UNIT-2

In the following slides we will consider what is involved in capturing a digital image of a real-world scene
- Image sensing and representation
- Image Acquisition
- Sampling and quantisation
- Resolution
- Basic relationship between pixels
- Linear and nonlinear operations

Before we discuss image acquisition recall that a digital image is composed of $M$ rows and $N$ columns of pixels each storing a value.
- Pixel values are most often grey levels in the range 0-255 (black-white)
- We will see later that images can easily be represented as matrices.

Images are typically generated by illuminating a scene and absorbing the energy reflected by the objects in that scene.
- Typical notions of illumination and scene can be way of:
  - X-rays of a skeleton
  - Ultrasound of an unborn baby
  - Electro-microscopic images of molecules

An image sensor is a device that converts an optical image into an electronic signal.
- It is used mostly in digital cameras, camera modules and other imaging devices.
- Early analog sensors were video camera tubes;
- Currently used types are:
  - semiconductor charge-coupled devices (CCD)
  - active pixel sensors in complementary metal-oxide-semiconductor (CMOS)
  - N-type metal-oxide-semiconductor (NMOS, Live MOS) technologies.

RGB Inside the Camera

Incoming Visible light

Visible Light passes through IR-blocking Filter

Color Filters control the color light reaching a sensor

Color blind sensors convert light reaching each sensor into electricity

IMAGE SENSORS-FLEX CIRCUIT ASSEMBLY

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7/28/2014
**IMAGE SENSING**

- Incoming energy lands on a sensor material responsive to that type of energy and this generates a voltage.
- There are 3 principal sensor arrangements (produce an electrical output proportional to light intensity).
- Collections of sensors are arranged to capture images.
  - (i) Single Imaging Sensor
  - (ii) Line sensor
  - (iii) Array sensor

**IMAGE ACQUISITION USING A SINGLE SENSOR**

- The most common sensor of this type is the photodiode, which is constructed of silicon materials and whose output voltage waveform is proportional to light.
- The use of a filter in front of a sensor improves selectivity.
- For example, a green (pass) filter in front of a light sensor favours light in the green band of the color spectrum.
- In order to generate a 2-D image using a single sensor, there have to be relative displacements in both the x- and y-directions between the sensor and the area to be imaged.

**IMAGE SENSING USING SENSOR STRIPS AND RINGS**

- The strip provides imaging elements in one direction.
- Motion perpendicular to the strip provides imaging in the other direction.
- This is the type of arrangement used in most flatbed scanners.
- Sensing devices with 4000 or more in-line sensors are possible.
- In-line sensors are used routinely in airborne imaging applications, in which the imaging system is mounted on an aircraft that flies at a constant altitude and speed over the geographical area to be imaged.

**IMAGE SENSING USING SENSOR ARRAYS**

- This type of arrangement is found in digital cameras.
- A typical sensor for these cameras is a CCD array, which can be manufactured with a broad range of sensing properties and can be packaged in rugged arrays of 4000 * 4000 elements or more.
- CCD sensors are used widely in digital cameras and other light sensing instruments.

**IMAGE SAMPLING AND QUANTISATION**

- A digital sensor can only measure a limited number of samples at a discrete set of energy levels.
- Quantisation is the process of converting a continuous analogue signal into a digital representation of the signal.
A Simple Image Formation Model

- Binary images: images having only two possible brightness levels (black and white)
- Gray scale images: "black and white" images
- Color images: can be described mathematically as three gray scale images

Let \( f(x, y) \) be an image function, then

\[
 f(x, y) = I(x, y) \cdot R(x, y),
\]

where \( I(x, y) \): the illumination function
\( R(x, y) \): the reflection function

Note: \( 0 < I(x, y) < \infty \) and \( 0 < R(x, y) < 1 \).

- For digital images the minimum gray level is usually 0, but the maximum depends on number of quantization levels used to digitize an image.
- The most common is 256 levels, so that the maximum level is 255.

Remember that a digital image is always only an approximation of a real world scene.
**SPATIAL RESOLUTION**

- The spatial resolution of an image is determined by how sampling was carried out.
- Spatial resolution simply refers to the smallest discernable detail in an image.

- Vision specialists will often talk about pixel size.
- Graphic designers will talk about dots per inch (DPI).
INTENSITY LEVEL RESOLUTION

- Intensity level resolution refers to the number of intensity levels used to represent the image
  - The more intensity levels used, the finer the level of detail discernable in an image
  - Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

<table>
<thead>
<tr>
<th>Number of Bits</th>
<th>Number of Intensity Levels</th>
<th>Examples</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0, 1</td>
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<tr>
<td>2</td>
<td>4</td>
<td>00, 01, 10, 11</td>
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<tr>
<td>4</td>
<td>16</td>
<td>0000, 0001, 0101, 1111</td>
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<tr>
<td>8</td>
<td>256</td>
<td>00110011, 01010101</td>
</tr>
<tr>
<td>16</td>
<td>65,536</td>
<td>1010101010101010</td>
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</table>

256 grey levels (8 bits per pixel)
128 grey levels (7 bits)
64 grey levels (6 bits)
32 grey levels (5 bits)
RESOLUTION: HOW MUCH IS ENOUGH?

- The big question with resolution is always: how much is enough?
- This all depends on what is in the image and what you would like to do with it.
- Key questions include:
  - Does the image look aesthetically pleasing?
  - Can you see what you need to see within the image?

RESOLUTION: HOW MUCH IS ENOUGH? (CONT...)

- The picture on the right is fine for counting the number of cars, but not for reading the number plate.
INTENSITY LEVEL RESOLUTION (CONT...)

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INTENSITY LEVEL RESOLUTION (CONT...)

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INTENSITY LEVEL RESOLUTION (CONT...)

SUMMARY
• We have looked at:
  • Human visual system
  • Light and the electromagnetic spectrum
  • Image representation
  • Image sensing and acquisition
  • Sampling, quantisation and resolution
• Next time we start to look at techniques for image enhancement

Aliasing and Moiré Pattern
• All signals (functions) can be shown to be made up of a linear
  combination sinusoidal signals (sines and cosines) of different
  frequencies.
• For physical reasons, there is a highest frequency component in all
  real world signals.
• Theoretically,
  • If a signal is sampled at more than twice its highest frequency component,
    then it can be reconstructed exactly from its samples.
  • But, if it is sampled at less than that frequency (called undersampling),
    then aliasing will result.
  • This causes frequencies to appear in the sampled signal that were not in
    the original signal.
• The Moiré pattern: The vertical low frequency pattern is a new frequency
  not in the original patterns.
Aliasing and Moiré Pattern

- Note that subsampling of a digital image will cause **undersampling** if the subsampling rate is less than twice the maximum frequency in the signal image.

- Aliasing can be prevented if a signal is filtered to eliminate high frequencies so that its highest frequency component will be less than twice the sampling rate.

- **Gating function**: exists for all space or time and has value zero everywhere except for a finite range of space/time. Often used for theoretical analysis of signals. But, a gating signal is mathematically defined and contains unbounded frequencies.

- A signal which is periodic, \( x(t) = x(t+T) \) for all \( t \) and where \( T \) is the period, has a finite maximum frequency component. So it is a **bandlimited signal**.

- Sampling at a higher sampling rate (usually twice or more) than necessary to prevent aliasing is called **oversampling**.

Zooming and Shrinking Digital Images

- **Zooming**: increasing the number of pixels in an image so that the image appears larger.
  - Nearest neighbor interpolation
  - **Bilinear interpolation**
  - Smoother
  - Higher order interpolation
  - Image shrinking: subsampling

Relationships Between Pixels

- Neighbors of a pixel
- There are three kinds of neighbors of a pixel:
  - \( N(p) \) 4-neighbors: the set of horizontal and vertical neighbors
  - \( N(p) \) diagonal neighbors: the set of 4 diagonal neighbors
  - \( N(p) \) 8-neighbors: union of 4-neighbors and diagonal neighbors

Some Basic Relationships Between Pixels
NEIGHBORS OF A PIXEL

- A pixel \(p\) at coordinates \((x, y)\) has four horizontal and vertical neighbors whose coordinates are given by:
  - \((x+1, y)\)
  - \((x-1, y)\)
  - \((x, y+1)\)
  - \((x, y-1)\)

This set of pixels, called the 4-neighbors of \(p\), is denoted by \(N_4(p)\). Each pixel is one unit distance from \((x, y)\) and some of the neighbors of \(p\) lie outside the digital image if \((x, y)\) is on the border of the image.

- The four diagonal neighbors of \(p\) have coordinates:
  - \((x+1, y+1)\)
  - \((x+1, y-1)\)
  - \((x-1, y+1)\)
  - \((x-1, y-1)\)

These points, together with the 4-neighbors, are called the 8-neighbors of \(p\), denoted by \(N_8(p)\).

ADJACENCY AND CONNECTIVITY

- Two pixels are adjacent if they are neighbors and their gray levels are similar.
- Let \(V\) a set of intensity values used to define adjacency and connectivity.
- In a binary image, \(V = \{1\}\), if we are referring to adjacency of pixels with value 1.
- In a gray-scale image, this idea is the same, but \(V\) typically contains more elements, for example, \(V = \{180, 181, 182, \ldots, 200\}\).
- If the possible intensity values 0 – 255, \(V\) set can be any subset of these 256 values.

TYPES OF ADJACENCY

1. **4-adjacency**: Two pixels \(p\) and \(q\) with values from \(V\) are 4-adjacent if \(q\) is in the set \(N_4(p)\).
2. **8-adjacency**: Two pixels \(p\) and \(q\) with values from \(V\) are 8-adjacent if \(q\) is in the set \(N_8(p)\).
3. **m-adjacency** (mixed)

Binary Image Adjacency Between Pixels

TYPES OF ADJACENCY

- **m-adjacency**: Two pixels \(p\) and \(q\) with values from \(V\) are m-adjacent if:
  - \(q\) is in \(N_4(p)\) or
  - \(q\) is in \(N_8(p)\) and the set \(N_4(p) \cap N_8(p)\) has no pixel whose values are from \(V\) (no intersection)

- Important Note: the type of adjacency used must be specified
TYPES OF ADJACENCY

- Mixed adjacency is a modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used.
- For example:

  ![Diagram of Mixed Adjacency](image)

  Figure 2.2a (a) Arrangement of pixels (b) pixels that are 8-adjacent (shown dashed) to the center pixel (c) m-adjacency.

- In this example, we can note that to connect between two pixels (finding a path between two pixels):
  - In 8-adjacency way, you can find multiple paths between two pixels
  - While, in m-adjacency, you can find only one path between two pixels
  - So, m-adjacency has eliminated the multiple path connection that has been generated by the 8-adjacency.

ADJACENCY: AN EXAMPLE

- 4-adjacency: Two pixels p and q with the values from set V are 4-adjacent if q is in the set of N_4(p).
  
  \[ V = \{0, 1\} \]

  - Example:
    
    \[
    \begin{array}{ccc}
    1 & 1 & 0 \\
    1 & 1 & 0 \\
    1 & 0 & 1 \\
    \end{array}
    \]

    p in RED color
    q can be any value in GREEN color.

- 8-adjacency: Two pixels p and q with the values from set V are 8-adjacent if q is in the set of N_8(p).
  
  \[ V = \{1, 2\} \]

  - Example:
    
    \[
    \begin{array}{ccc}
    0 & 1 & 1 \\
    0 & 2 & 0 \\
    0 & 0 & 1 \\
    \end{array}
    \]

    p in RED color
    q can be any value in GREEN color.
**ADJACENCY, CONNECTIVITY**

**m-adjacency:** Two pixels p and q with the values from set \( V \) are m-adjacent if
(i) \( q \) is in \( N_4(p) \) OR
(ii) \( q \) is in \( N_D(p) \) & the set \( N_4(p) \cap N_4(q) \) have no pixels whose values are from \( V \).

\[ e.g. V = \{1\} \]

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**Soln:** b & c are m-adjacent.

**ADJACENCY, CONNECTIVITY**

**m-adjacency:** Two pixels p and q with the values from set \( V \) are m-adjacent if
(i) \( q \) is in \( N_4(p) \)
(ii) b & c

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**Soln:** b & e are m-adjacent.

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**Soln:** b & e are m-adjacent.
ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set V are m-adjacent if
(i) q is in N_4(p) & the set N_4(p) ∩ N_4(q) have no pixels whose values are from V.
e.g. V = {1}
(ii) e & i

Solving: e & i are m-adjacent.

SUBSET ADJACENCY

Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2.
Adjacent means, either 4-, 8- or m-adjacency.
Example:
For V=1, Determine whether these two subsets are
i) 4 Adjacent
ii) 8 Adjacent
iii) M-adjacent

A DIGITAL PATH

A digital path (or curve) from pixel p with coordinate (x,y) to pixel q with coordinate (s,t) is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1), ...
(x_n,y_n) where (x_0,y_0) = (x,y) and (x_n,y_n) = (s,t) and pixels (x_i,y_i) and (x_{i+1},y_{i+1}) are adjacent for 1 ≤ i ≤ n.

n is the length of the path.
If (x_n,y_n) = (x_0,y_0), the path is closed.
We can specify 4-, 8- or m-paths depending on the type of adjacency specified.
A DIGITAL PATH

• Return to the previous example:

In figure (b) the paths between the top right and bottom right pixels are 8-paths. And the path between the same 2 pixels in figure (c) is m-path.

CONNECTIVITY

• Let $S$ represent a subset of pixels in an image, two pixels $p$ and $q$ are said to be connected in $S$ if there exists a path between them consisting entirely of pixels in $S$.

• For any pixel $p$ in $S$, the set of pixels that are connected to it in $S$ is called a connected component of $S$. If it only has one connected component, then set $S$ is called a connected set.

PATHS

Example #1: Consider the image segment shown in figure. Compute length of the shortest-4, shortest-8 & shortest-m paths between pixels $p$ & $q$ where $V = \{1, 2\}$.

<table>
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<th>2</th>
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<td>p</td>
<td>2</td>
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PATHS

Example #1:
Shortest-4 path:

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<td>2</td>
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PATHS

Example #1:
Shortest-4 path:

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<td>p</td>
<td>2</td>
<td>1</td>
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</table>
Example #1:
Shortest 4 path:
V = \{1, 2\}.
\begin{align*}
4 & 2 & 3 & 2 q \\
3 & 3 & 1 & 3 \\
2 & 3 & 2 & 2 \\
p & 1 & 2 & 3 \\
\end{align*}

So, Path does not exist.

Example #1:
Shortest 8 path:
V = \{1, 2\}.
\begin{align*}
4 & 2 & 3 & 2 q \\
3 & 3 & 1 & 3 \\
2 & 3 & 2 & 2 \\
p & 1 & 2 & 3 \\
\end{align*}
**Example #1:** Shortest path: $V = \{1, 2\}$.

So, shortest path = 4.
Example # 1:
Shortest - m path:
V = \{1, 2\}.

\begin{array}{ccc}
4 & 2 & 3 \\
3 & 3 & 1 \\
2 & 3 & 2 \\
p & 2 & 2 \\
\end{array}

\begin{array}{ccc}
2 & q & 3 \\
3 & 3 & 1 \\
2 & 3 & 2 \\
p & 2 & 2 \\
\end{array}

So, shortest - m path = 5

Example # 2:
Find Shortest 4, 6, - m path:
V = \{3, 2\}.

\begin{array}{ccc}
4 & 2 & 3 \\
3 & 3 & 1 \\
2 & 3 & 2 \\
p & 2 & 2 \\
\end{array}

\begin{array}{ccc}
2 & q & 3 \\
3 & 3 & 1 \\
2 & 3 & 2 \\
p & 2 & 2 \\
\end{array}

- Region
  - Let R be a subset of pixels in an image
  - R is called a region if every pixel in R is connected to any other pixel in R.

- Boundary
  The boundary (also called border or contour) of a region R is the set of pixels in the region that have one or more neighbors that are not in R.
REGION AND BOUNDARY

If \( R \) happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns in the image.

This extra definition is required because an image has no neighbors beyond its borders.

Normally, when we refer to a region, we are referring to a subset of an image, and any pixels in the boundary of the region that happen to coincide with the border of the image are included implicitly as part of the region boundary.

REGIONS & BOUNDARIES

Regions that are not adjacent are said to be disjoint.

We consider 4- and 8-adjacency when referring to regions.

Below regions are adjacent only if 8-adjacency is used.

```
 1 1 1
1 0 1 R_i
0 1 0
0 0 1
1 1 1 R_j
1 1 1
```
**CITY BLOCK DISTANCE OR $D_4$ DISTANCE**

- The $D_4$ distance (also called city-block distance) between $p$ and $q$ is defined as:
  $$D_4(p, q) = |x - s| + |y - t|$$

Pixels having a $D_4$ distance from $(x, y)$, less than or equal to some value $r$ form a diamond centered at $(x, y)$.

**DISTANCE MEASURES - $D_4$**

**Example:**
The pixels with distance $D_4 \leq 2$ from $(x, y)$ form the following contours of constant distance.

The pixels with $D_4 = 1$ are the 4-neighbors of $(x, y)$.

**CHESSBOARD DISTANCE OR $D_8$ DISTANCE**

- The $D_8$ distance (also called chessboard distance) between $p$ and $q$ is defined as:
  $$D_8(p, q) = \max( |x - s|, |y - t|)$$

Pixels having a $D_8$ distance from $(x, y)$, less than or equal to some value $r$ form a square centered at $(x, y)$.

**DISTANCE MEASURES - $D_8$**

**Example:**
$D_8$ distance $\leq 2$ from $(x, y)$ form the following contours of constant distance.

**$D_m$ DISTANCE**

- $D_m$ distance:
  is defined as the shortest $m$-path between the points.
In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.
DISTANCE MEASURES

- Example:
  - Consider the following arrangement of pixels and assume that \( p_1, p_2, \) and \( p_4 \) have value 1 and that \( p_1 \) and \( p_2 \) can have can have a value of 0 or 1.
  - Suppose that we consider the adjacency of pixels values 1 (i.e. \( V = \{1\} \)).

\[
\begin{array}{c}
\text{P}_3 & \text{P}_4 \\
\text{P}_1 & \text{P}_2 \\
\text{P} \\
\end{array}
\]

DISTANCE MEASURES

- Cont. Example:
  Now, to compute the \( D_m \) between points \( p \) and \( p_4 \).
Here we have 4 cases:
  - Case 1: \( p_1 = 0 \) and \( p_3 = 0 \)
    The length of the shortest \( m \)-path (the \( D_m \) distance) is 2 (\( p, p_2, p_4 \)).

\[
\begin{array}{c}
\text{P}_3 & \text{P}_4 \\
\text{P}_1 & \text{P}_2 \\
\text{P} \\
\end{array}
\]

DISTANCE MEASURES

- Cont. Example:
  Case 2: \( p_1 = 1 \) and \( p_3 = 0 \)
  now, \( p_2 \) and \( p \) will no longer be adjacent (see \( m \)-adjacency definition)
  then, the length of the shortest
  path will be 3 (\( p, p_1, p_2, p_4 \)).

\[
\begin{array}{c}
\text{P}_3 & \text{P}_4 \\
\text{P}_1 & \text{P}_2 \\
\text{P} \\
\end{array}
\]

DISTANCE MEASURES

- Cont. Example:
  Case 3: \( p_1 = 0 \) and \( p_3 = 1 \)
  The same applies here, and the shortest \( m \)-path will be 3 (\( p, p_2, p_3, p_4 \)).

\[
\begin{array}{c}
\text{P}_3 & \text{P}_4 \\
\text{P}_1 & \text{P}_2 \\
\text{P} \\
\end{array}
\]

DISTANCE MEASURES

- Cont. Example:
  Case 4: \( p_1 = 1 \) and \( p_3 = 1 \)
  The length of the shortest \( m \)-path will be 4 (\( p, p_1, p_2, p_3, p_4 \)).

\[
\begin{array}{c}
\text{P}_3 & \text{P}_4 \\
\text{P}_1 & \text{P}_2 \\
\text{P} \\
\end{array}
\]

LINEAR & NON-LINEAR OPERATIONS

\[ H(af+bh)=aH(f)+bH(g) \]

- Where \( H \) is an operator whose input and output are images.
- \( f \) and \( g \) are two images \( a \) and \( b \) are constants.

- \( H \) is said to be linear operation if it satisfies the above equation or else \( H \) is a
  non-linear operator.
A Pixel p at coordinates (x, y) has 4 horizontal and vertical neighbors.

Their coordinates are given by:
- (x+1, y)
- (x-1, y)
- (x, y+1)
- (x, y-1)

This set of pixels is called the \( N_4(p) \) neighbors of p.

They are denoted by \( N_8(p) \).

They are denoted by \( N_{4,8}(p) \).

In binary images, 2 pixels are adjacent if they are neighbors & have some intensity values either 0 or 1.

In gray scale, image contains more gray level values in range 0 to 255.
ADJACENCY, CONNECTIVITY

Connectivity: 2 pixels are said to be connected if there exists a path between them.

Let S represent a subset of pixels in an image.

Two pixels p & q are said to be connected in S if there exists a path between them consisting entirely of pixels in S.

For any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S.

PATHS

Paths: A path from pixel p with coordinate \((x, y)\) with pixel q with coordinate \((s, t)\) is a sequence of distinct sequence with coordinates \((x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\) where

\[ (x, y) = (x_0, y_0) \]
\[ & (s, t) = (x_n, y_n) \]

Closed path: \((x_0, y_0) = (x_n, y_n)\)

DISTANCE MEASURES

Distance Measures: Distance between pixels p, q & z with co-ordinates \((x, y), (s, t)\) & \((v, w)\) resp. is given by:

a) \[ D(p, q) \geq 0 \quad [D(p, q) = 0 \text{ if } p = q] \]
   called reflexivity

b) \[ D(p, q) = D(q, p) \]
   called symmetry

c) \[ D(p, z) \leq D(p, q) + D(q, z) \]
   called transitivity

Euclidean distance between p & q is defined as:

\[ D_e(p, q) = \sqrt{(x - s)^2 + (y - t)^2} \]

DISTANCE MEASURES

City Block Distance: The \(D_4\) distance between p & q is defined as

\[ D_4(p, q) = |x - s| + |y - t| \]

In this case, pixels having \(D_4\) distance from \((x, y)\) less than or equal to some value \(r\) form a diamond centered at \((x, y)\).

\[
\begin{array}{cccccc}
2 & 1 & 2 & 1 & 2 & 2 \\
1 & 0 & 1 & 2 & 2 & 1 \\
0 & 1 & 1 & 2 & 2 & 2 \\
1 & 1 & 1 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 \\
\end{array}
\]

Pixels with \(D_4\) distance \(\leq 2\) forms the following contour of constant distance.

DISTANCE MEASURES

Chess-Board Distance: The \(D_8\) distance between p & q is defined as

\[ D_8(p, q) = \max(|x - s|, |y - t|) \]

In this case, pixels having \(D_8\) distance from \((x, y)\) less than or equal to some value \(r\) form a square centered at \((x, y)\).

\[
\begin{array}{cccccc}
2 & 1 & 2 & 1 & 2 & 2 \\
1 & 0 & 1 & 2 & 2 & 1 \\
0 & 1 & 1 & 2 & 2 & 2 \\
1 & 1 & 1 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 \\
\end{array}
\]

Pixels with \(D_8\) distance \(\leq 2\) forms the following contour of constant distance.

End of Unit 2