Digital Image Processing
Module 5:
Image Segmentation

Importance of Image Segmentation

Image segmentation is used to separate an image into constituent parts based on some image attributes. Image segmentation is an important step in image analysis.

Benefit
1. Image segmentation reduces huge amount of unnecessary data while retaining only importance data for image analysis.
2. Image segmentation converts bitmap data into better structured data which is easier to be interpreted.

Image Attributes for Image Segmentation

1. Similarity properties of pixels inside the object are used to group pixels into the same set.
2. Discontinuity of pixel properties at the boundary between object and background is used to distinguish between pixels belonging to the object and those of background.

Spatial Filtering Application to Shape Detection

- One application of spatial filtering is shape detection: finding locations of objects with the desired shape.
- Unlike frequency selective masks that are designed based on the concept of frequency, shape detection masks are derived from the shapes to be detected themselves.
- A mask for shape detection usually contains the shape or a part of the shape to be detected.
- The location that is most correlated to the mask is the location where the highest filter response occurs. The shape is most likely to exist there.

Point Detection

- We can use Laplacian masks for point detection.
- Laplacian masks have the largest coefficient at the center of the mask while neighbor pixels have an opposite sign.
- This mask will give the high response to the object that has the similar shape as the mask such as isolated points.
- Notice that sum of all coefficients of the mask is equal to zero. This is due to the need that the response of the filter must be zero inside a constant intensity area.

Element of Image Analysis

Preprocess
Image acquisition, restoration, and enhancement

Intermediate process
Image segmentation and feature extraction

High level process
Image interpretation and recognition
**Point Detection**

Point detection can be done by applying the thresholding function:

\[
g(x, y) = \begin{cases} 
1 & \text{if } |f(x, y)| \geq T \\
0 & \text{otherwise}
\end{cases}
\]

![X-ray image of the turbine blade with porosity](Image)

![Laplacian image](Image)

![After thresholding](Image)

**Line Detection**

- Similar to point detection, line detection can be performed using the mask that has the shape similar to a part of a line.
- There are several directions that the line in a digital image can be.
- For a simple line detection, 4 directions that are mostly used are horizontal, +45 degree, vertical and –45 degree.

![Line detection masks](Image)

**Line Detection Example**

- Binary wire bond mask image
- Absolute value of result after processing with -45 line detector
- Result after thresholding

![Notice that -45 degree lines are most sensitive](Image)

**Edges**

- Ideally, objects and background have different intensities. Therefore, edges of the objects are the areas where abrupt intensity changes occur.

![Gray level profile of a horizontal line through the image](Image)

![Gray level profile of a horizontal line through the image](Image)

**Smoothed Step Edge and Its Derivatives**

- **Gray level profile**
- **The 1st derivative**
- **The 2nd derivative**
Derivative Based Edge Detection

- From the previous slide, we can conclude that:
  Local maxima of the absolute of the 1st derivative and Zero crossing of the 2nd derivative occur at edges.
- Therefore, for detecting edges, we can apply zero crossing detection to the 2nd derivative image or thresholding the absolute of the 1st derivative image.
- Nevertheless, derivative operator is very sensitive to noise as we will see in the next slide.

Noisy Edges and Derivatives

Derivative operator is a highpass filter and thus enhances noise.
Edge responses are buried by noise.

Masks for Estimating Partial Derivatives

Normally, the mask for estimating partial derivative is anti-symmetry with respect to the orthogonal axis

For example, the Sobel mask for computing $\frac{df}{dx}$ is anti-symmetry with respect to the y-axis. It has the positive sign on the right side and negative sign on the left side.

Notice that sum of all coefficients is equal to zero to make sure that the response of a constant intensity area is zero.

Masks for Detecting Diagonal Edges

The mask for detecting -45-degree edges is anti-symmetry with respect to the -45-degree lines while the mask for detecting 45-degree edges is anti-symmetry with respect to the 45-degree lines.

Detection of Discontinuities

An image gradient is a directional change in the intensity or color in an image.

- First-order derivatives:
  - The gradient of an image $f(x,y)$ at location $(x,y)$ is defined as the vector:
    $$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{df}{dx} \\ \frac{df}{dy} \end{bmatrix}$$
  - The magnitude of this vector: $Vf = \text{mag}(\nabla f) = \sqrt{G_x^2 + G_y^2}$
  - The direction of this vector: $\alpha(x, y) = \tan^{-1} \left( \frac{G_x}{G_y} \right)$
Detection of Discontinuities
Gradient Operators

Prewitt masks for detecting diagonal edges

\[
\begin{array}{ccc}
0 & 1 & 1 \\
-1 & 0 & 1 \\
0 & -1 & 1 \\
\end{array}
\]

Sobel masks for detecting diagonal edges

\[
\begin{array}{ccc}
0 & 1 & 2 \\
-2 & 0 & -1 \\
-1 & -1 & 1 \\
\end{array}
\]

Figure 10.9 Prewitt and Sobel masks for detecting diagonal edges.

Example of Image Gradient

\[ f(x, y) \]

Example of Diagonal Edges

Using -45-degree mask

\[
\begin{array}{ccc}
0 & 1 & 2 \\
-1 & 1 & 0 \\
0 & -1 & 0 \\
\end{array}
\]

Using 45-degree mask

\[
\begin{array}{ccc}
0 & -1 & 1 \\
0 & 1 & -1 \\
1 & -1 & 0 \\
\end{array}
\]

Note: the original image is smoothed by a 5x5 moving average mask first.

Detection of Discontinuities
Gradient Operators

- Second-order derivatives: (The Laplacian)
  - The Laplacian of a 2D function \( f(x,y) \) is defined as
    \[
    \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}
    \]
  - Two forms in practice:

Laplacian Masks

The Laplacian masks are used to estimate the Laplacian image:

\[
\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}
\]

Ideally, the Laplacian mask must be directional invariant: symmetry in all direction (radially symmetry). However, for 3x3 masks, there are Only 8 possible directions. Hence, we can use the following masks:

\[
\begin{array}{cccc}
0 & -1 & 0 & -1 \\
-1 & 0 & 1 & -1 \\
0 & -1 & 0 & -1 \\
\end{array}
\]
Detection of Discontinuities
Gradient Operators

- Consider the function:
  \[ h(r) = -e^{-\frac{r^2}{2\sigma^2}} \]
  where \( r^2 = x^2 + y^2 \)
  and \( \sigma \): the standard deviation

- The Laplacian of \( h \) is
  \[ \nabla^2 h(r) = \left[ \frac{r^2 - \sigma^2}{\sigma^4} \right] e^{-\frac{r^2}{2\sigma^2}} \]

- The Laplacian of a Gaussian sometimes is called the "Mexican hat function." It also can be computed by smoothing the image with the Gaussian smoothing mask, followed by application of the Laplacian mask.

Example of Laplacian Image

The angiogram image (blood vessels)

We can compute the Laplacian image by:
1. Smooth the image by the Gaussian mask
2. Compute the Laplacian image using the mask

Laplacian Masks

For a large scale Laplacian mask, we can use a Laplacian of Gaussian (LOG) as a mask:

\[ \nabla^2 G(x, y) = \frac{1}{\pi \sigma^4} \left[ \frac{x^2 + y^2 - \sigma^2}{\sigma^4} \right] e^{-\frac{x^2 + y^2}{2\sigma^2}} \]

Thresholding

- Assumption: the range of intensity levels covered by objects of interest is different from the background.

\[ g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \\ 0 & \text{if } f(x, y) \leq T \end{cases} \]

Nonuniform Illumination Problem

An image can be expressed as

\[ f(x, y) = i(x, y) r(x, y) \]

where \( i(x, y) \): illumination function
\( r(x, y) \): reflectance component
Nonuniform Illumination and Global Thresholding

Global threshold level

Histogram

Global thresholding of nonuniform illumination image can cause huge errors!

Nonuniform illumination image

Global thresholding result

Nonuniform Illumination and Local Thresholding

Local thresholding:
1. Divide an image into subimages.
2. Threshold each subimage independently
   2.1 Compute histogram of each subimage and select a suitable threshold value for each subimage
   2.2 Threshold each subimage using a threshold value in 2.1
   2.3 Combine all local thresholding results

16 subimages

Result of local thresholding

Thresholding
Basic Adaptive Thresholding

How to solve this problem?

Answer: subdivision

Thresholding
Basic Adaptive Thresholding

Figure 10.30 (a) (b) (c) (d) (e) (f) Corresponding histograms of the improperly segmented subimage at top left. (f) Result of adaptively segmenting (d).
If areas of object and background are nearly equal, a histogram will be bimodal.

If areas of object and background are not balanced, a histogram will be unimodal.

**Histogram of Subimages and Local Thresholding**

**Optimum Thresholding**

Object

Background

\[ p_1(z) = \text{PDF of object pixels} \]

\[ p_2(z) = \text{PDF of background pixels} \]

Error due to background pixels classified as object pixels is:

\[ E_1(T) = \int_{-\infty}^{T} p_2(z) \, dz \]

Error due to object pixels classified as background pixels is:

\[ E_2(T) = \int_{T}^{\infty} p_1(z) \, dz \]

Total error:

\[ E(T) = P_1 E_1(T) + P_2 E_2(T) \]

- \[ P_1 = \text{Probability of occurrence of object pixels} \]
- \[ P_2 = \text{Probability of occurrence of background pixels} \]

**Thresholding: Optimal Global and Adaptive Thresholding**

- This method treats pixel values as probability density functions.
- The goal of this method is to minimize the probability of misclassifying pixels as either object or background.
- There are two kinds of error:
  - mislabeling an object pixel as background, and
  - mislabeling a background pixel as object.

**Region-Based Segmentation**

- Edges and thresholds sometimes do not give good results for segmentation.
- Region-based segmentation is based on the connectivity of similar pixels in a region.
  - Each region must be uniform.
  - Connectivity of the pixels within the region is very important.
- There are two main approaches to region-based segmentation: *region growing* and *region splitting*.
Basic Formulation

Let R represent the entire image region. Segmentation partitions R into n subregions, \( R_1, R_2, \ldots, R_n \), such that:

- a) Every pixel must be in a region
- b) Points in a region must be connected.
- c) Regions must be disjoint.
- d) All pixels in a region satisfy specific properties.
- e) Different regions have different properties.

\[ \bigcup_{i=1}^{n} R_i = R \]

Region growing

- Groups pixels into larger regions.
- Starts with a seed region.
- Grows region by merging neighboring pixels.

Iterative process
- How to start?
- How to iterate?
- When to stop?

Region-Based Segmentation

Region Growing

- Fig. 10.41 shows the histogram of Fig. 10.40 (a). It is difficult to segment the defects by thresholding methods. (Applying region growing methods are better in this case.)

How do we form regions?

- Region Growing
- Region Merging
- Region Splitting
- Split and Merge
- Watershed

What a computer sees
Region-Based Segmentation
Region Splitting and Merging

- Region splitting is the opposite of region growing.
  - First there is a large region (possibly the entire image).
  - Then a predicate (measurement) is used to determine if the region is uniform.
  - If not, then the method requires that the region be split into two regions.
  - Then each of these two regions is independently tested by the predicate (measurement).
  - This procedure continues until all resulting regions are uniform.

Region splitting

- Algorithm
  - One initial set that includes the whole image.
  - Similarity criteria.
  - Iteratively split regions into sub-regions.
  - Stop when no more splittings are possible.

Region merging

- Algorithm
  - Divide image into an initial set of regions.
    - One region per pixel.
  - Define a similarity criteria for merging regions.
  - Merge similar regions.
  - Repeat previous step until no more merge operations are possible.

The Watershed Transform

- Geographical inspiration.
  - Shed water over rugged terrain.
  - Each lake corresponds to a region.
- Characteristics
  - Computationally complex.
  - Great flexibility in segmentation.
  - Risk of over-segmentation.
The Drainage Analogy

- Two points are in the same region if they drain to the same point.

The Immersion Analogy

- The concept of watersheds is based on visualizing an image in three dimensions: two spatial coordinates versus gray levels.
- In such a topographic interpretation, we consider three types of points:
  - (a) points belonging to a regional minimum
  - (b) points at which a drop of water would fall with certainty to a single minimum
  - (c) points at which water would be equally likely to fall to more than one such minimum
- The principal objective of segmentation algorithms based on these concepts is to find the watershed lines.