Chapter-3

Waveform Coding Techniques

PCM [Pulse Code Modulation]

PCM is an important method of analog –to-digital conversion. In this modulation the analog signal is converted into an electrical waveform of two or more levels. A simple two level waveform is shown in fig 3.1.

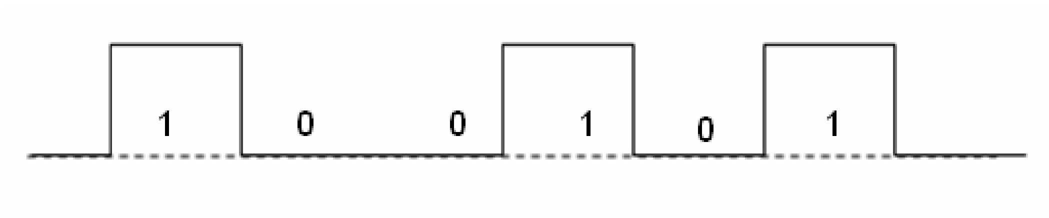

Fig:3.1 A simple binary PCM waveform

The PCM system block diagram is shown in fig 3.2. The essential operations in the transmitter of a PCM system are Sampling, Quantizing and Coding. The Quantizing and encoding operations are usually performed by the same circuit, normally referred to as analog to digital converter.

The essential operations in the receiver are regeneration, decoding and demodulation of the quantized samples. Regenerative repeaters are used to reconstruct the transmitted sequence of coded pulses in order to combat the accumulated effects of signal distortion and noise.

PCM Transmitter:

Basic Blocks:

1. Anti aliasing Filter
2. Sampler
3. Quantizer
4. Encoder

An anti-aliasing filter is basically a filter used to ensure that the input signal to sampler is free from the unwanted frequency components.
For most of the applications these are low-pass filters. It removes the frequency components of the signal which are above the cutoff frequency of the filter. The cutoff frequency of the filter is chosen such it is very close to the highest frequency component of the signal.

Sampler unit samples the input signal and these samples are then fed to the Quantizer which outputs the quantized values for each of the samples. The quantizer output is fed to an encoder which generates the binary code for every sample. The quantizer and encoder together is called as analog to digital converter.

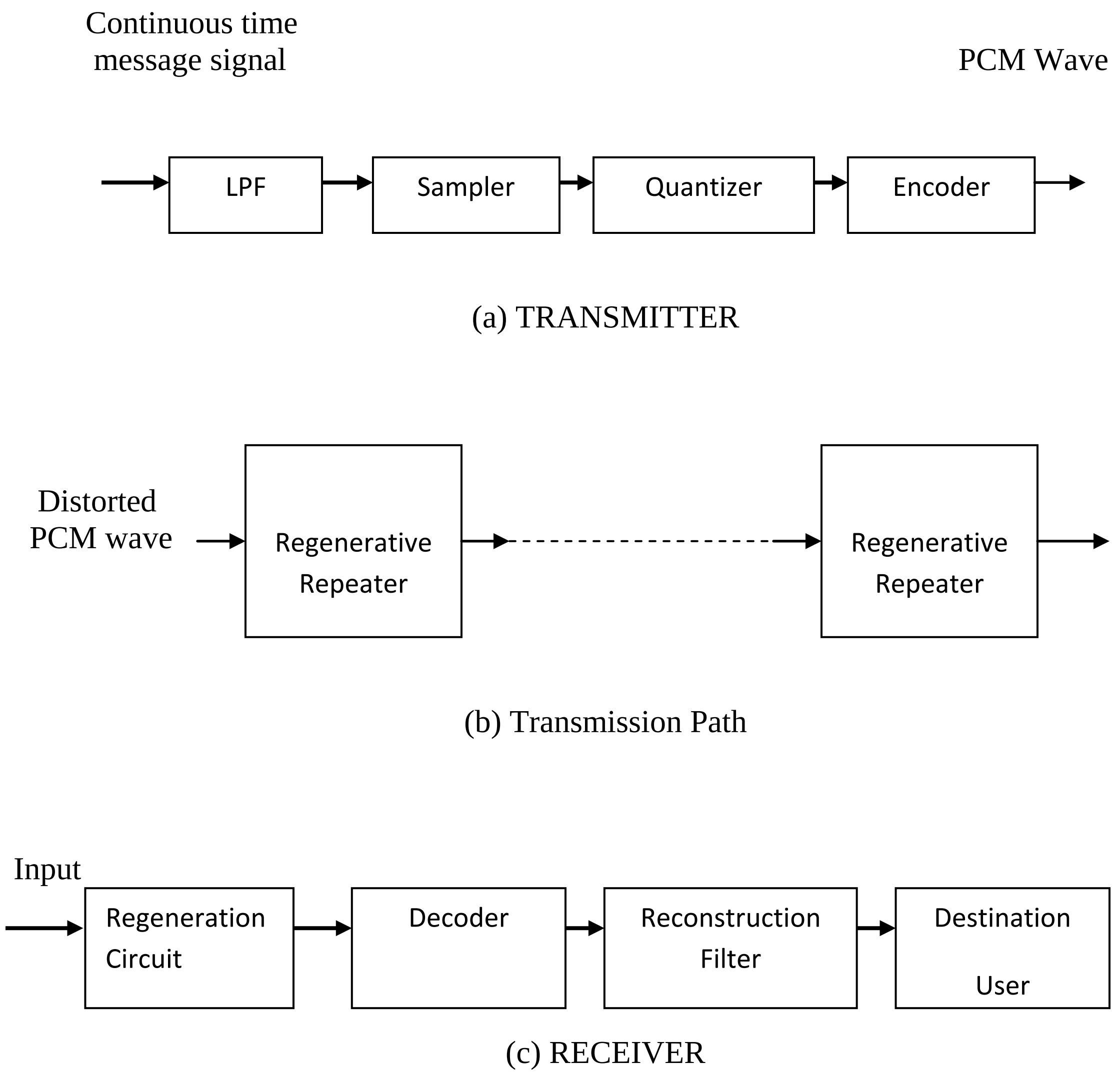

Fig: 3.2 - PCM System : Basic Block Diagram

REGENERATIVE REPEATER

REGENERATION: The feature of the PCM systems lies in the ability to control the effects of distortion and noise produced by transmitting a PCM wave through a channel. This is accomplished by reconstructing the PCM wave by means of regenerative repeaters.

Three basic functions: Equalization
Timing and
Decision Making

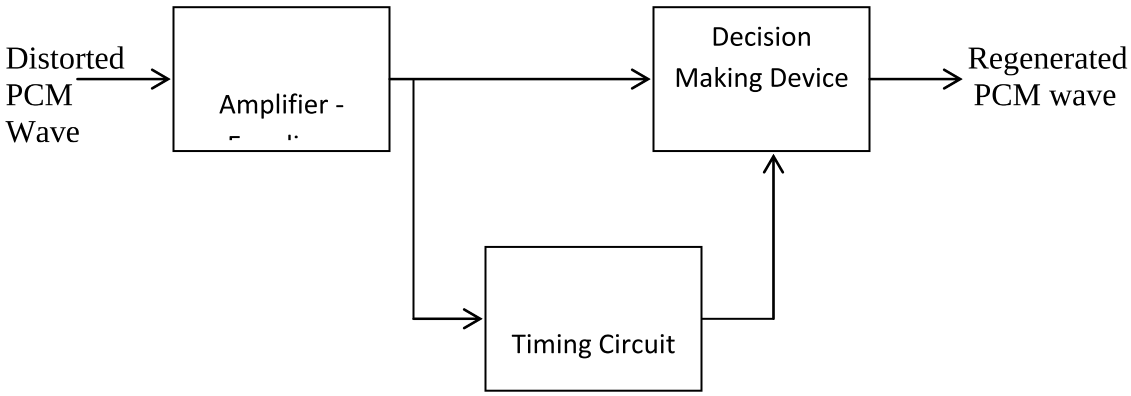

Fig: 3.3 - Block diagram of a regenerative repeater.

The equalizer shapes the received pulses so as to compensate for the effects of amplitude and phase distortions produced by the transmission characteristics of the channel.

The timing circuit provides a periodic pulse train, derived from the received pulses, for sampling the equalized pulses at the instants of time where the signal to noise ratio is maximum.

The decision device is enabled at the sampling times determined by the timing circuit. It makes it's decision based on whether the amplitude of the quantized pulse plus noise exceeds a predetermined voltage level.

Quantization Process:

The process of transforming Sampled amplitude values of a message signal into a discrete amplitude value is referred to as Quantization.

The quantization Process has a two-fold effect:

1. the peak-to-peak range of the input sample values is subdivided into a finite set of decision levels or decision thresholds that are aligned with the risers of the staircase, and
2. the output is assigned a discrete value selected from a finite set of representation levels that are aligned with the treads of the staircase..

A quantizer is memory less in that the quantizer output is determined only by the value of a corresponding input sample, independently of earlier analog samples applied to the input.

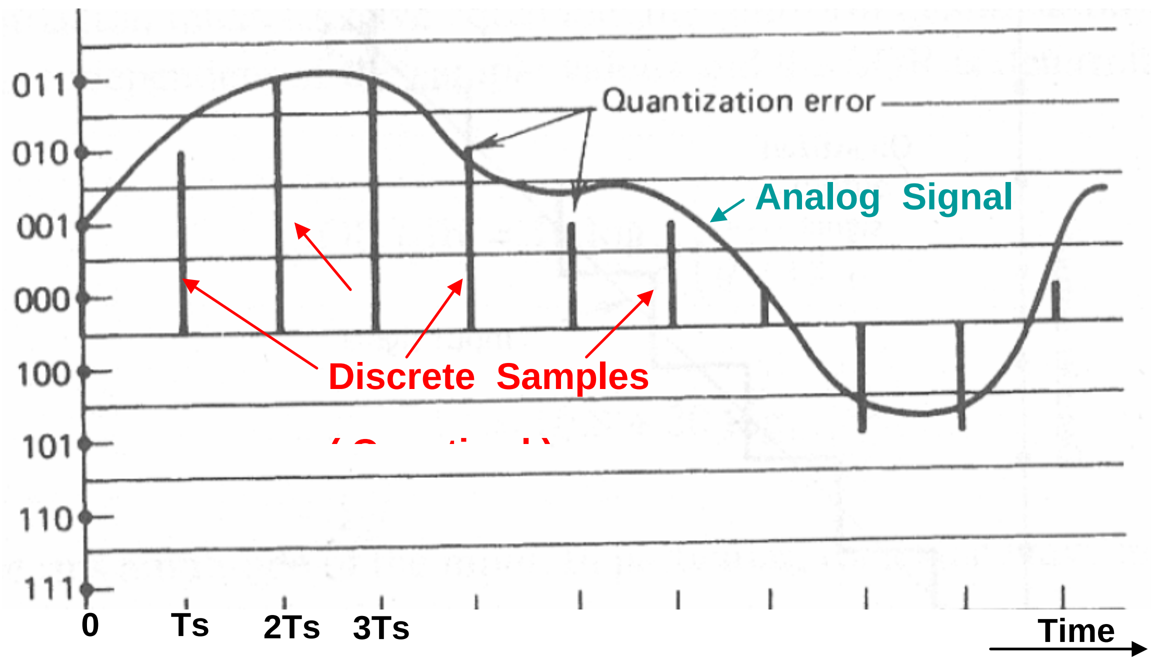

Fig:3.4   Typical Quantization process.

Types of Quantizers:

1. Uniform Quantizer
2. Non- Uniform Quantizer

In Uniform type, the quantization levels are uniformly spaced, whereas in non-uniform type the spacing between the levels will be unequal and mostly the relation is logarithmic.

Types of   Uniform Quantizers: (based on I/P - O/P Characteristics)

1. Mid-Rise type Quantizer
2. Mid-Tread type Quantizer

In the stair case like graph, the origin lies the middle of the tread portion in Mid –Tread type where as the origin lies in the middle of the rise portion in the Mid-Rise type.

Mid – tread type: Quantization levels – odd number.
Mid – Rise type: Quantization levels – even number.

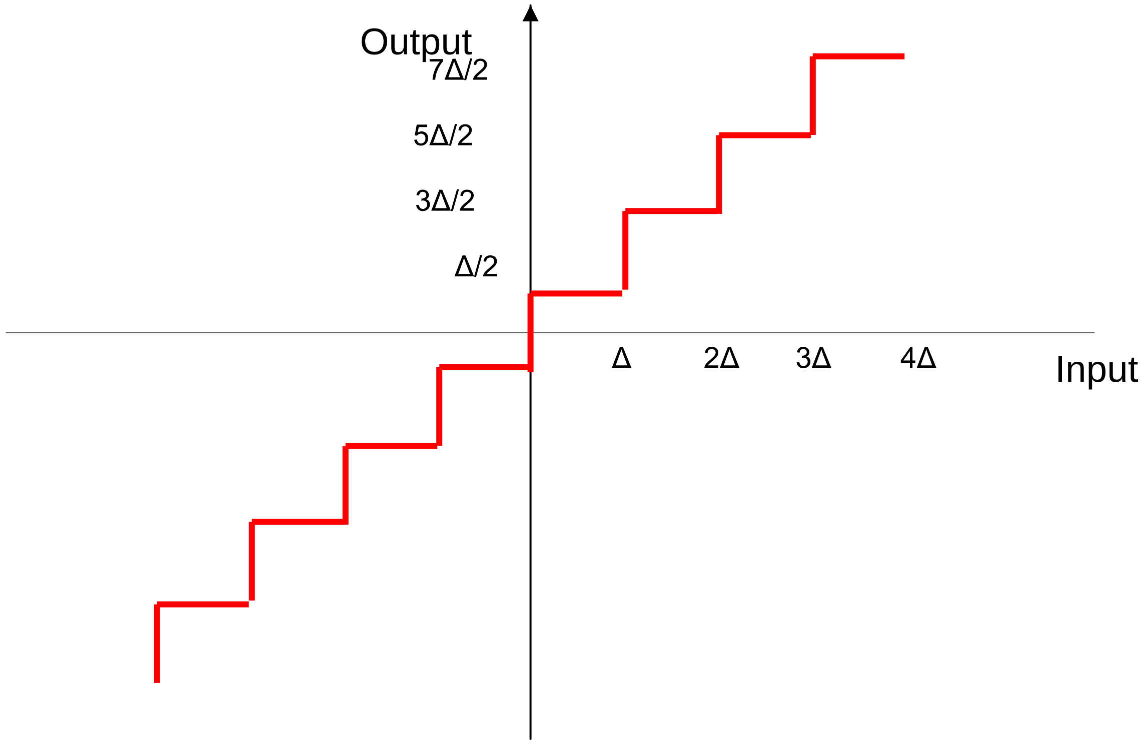

Fig:3.5 Input-Output Characteristics of a Mid-Rise type Quantizer

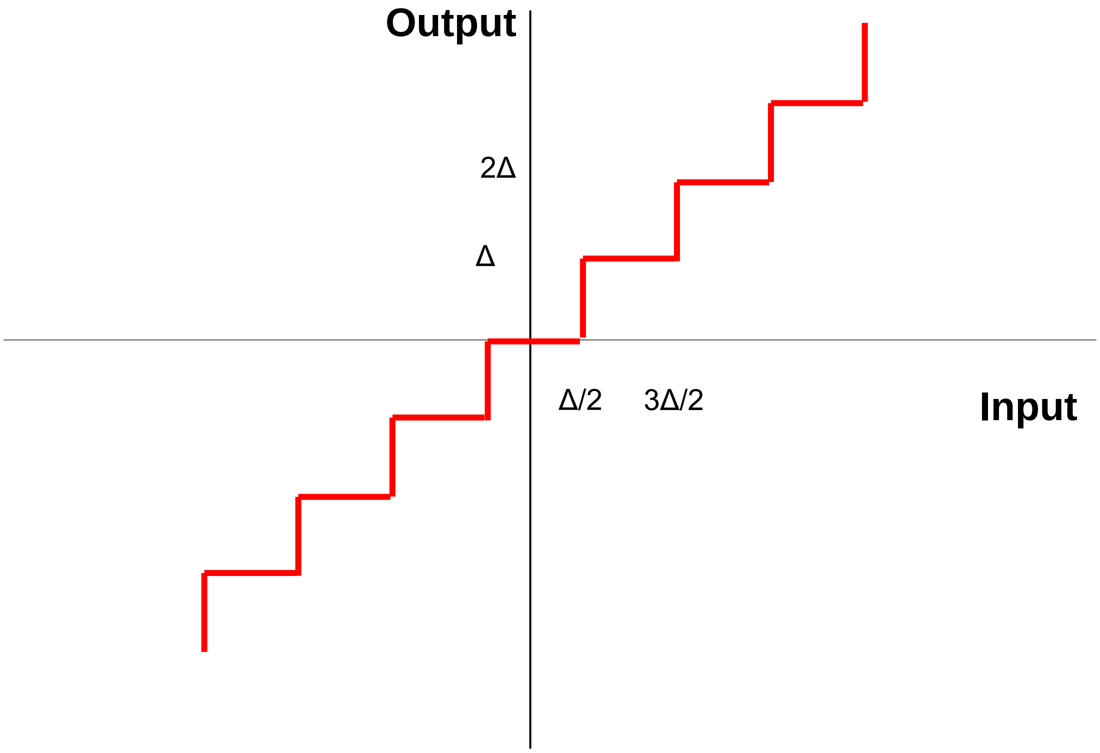

Fig:3.6 Input-Output Characteristics of a Mid-Tread type Quantizer

Quantization Noise and Signal-to-Noise:

"The Quantization process introduces an error defined as the difference between the input signal, x(t) and the output signal, yt). This error is called the Quantization Noise."

$$q(t) = x(t) - y(t)$$

Quantization noise is produced in the transmitter end of a PCM system by rounding off sample values of an analog base-band signal to the nearest permissible representation levels of the quantizer. As such quantization noise differs from channel noise in that it is signal dependent.

Let 'Δ' be the step size of a quantizer and L be the total number of quantization levels.

Quantization levels are 0, ± Δ., ± 2 Δ., ±3 Δ . . . .

The Quantization error, Q is a random variable and will have its sample values bounded by $[-(\Delta/2) < q < (\Delta/2)]$. If Δ is small, the quantization error can be assumed to a uniformly distributed random variable.

Consider a memory less quantizer that is both uniform and symmetric.

L = Number of quantization levels

X = Quantizer input

Y = Quantizer output

The output y is given by
Y=Q(x) ------------- (3.1)

which is a staircase function that befits the type of mid tread or mid riser quantizer of interest.

Suppose that the input ‘x’ lies inside the interval

$$I_k = \{x_k < x \le x_{k+1}\} \qquad k = 1,2,\text{---------}L \qquad \text{------- } (3.2)$$

where x_k and x_{k+1} are decision thresholds of the interval I_k as shown in figure 3.7.

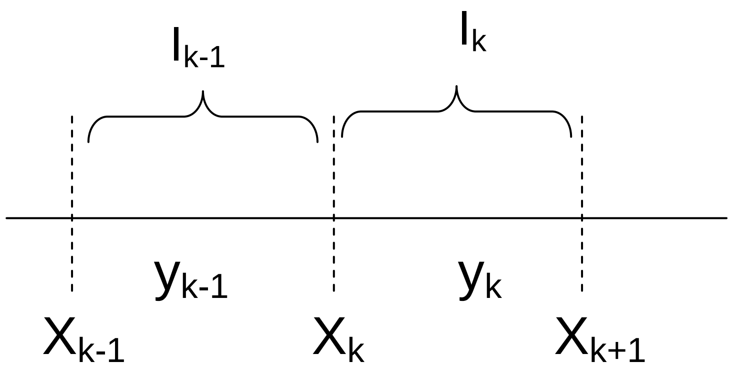

Fig:3.7 Decision thresholds of the equalizer

Correspondingly, the quantizer output y takes on a discrete value

$$Y = y_k \quad \text{if x lies in the interval } I_k$$

Let q = quantization error with values in the range $-\Delta/2 \le q \le \Delta/2$ then

$$Y_k = x+q \quad \text{if ‘n’ lies in the interval } I_k$$

Assuming that the quantizer input ‘n’ is the sample value of a random variable ‘X’ of zero mean with variance σx^2.

The quantization noise uniformly distributed through out the signal band, its interfering effect on a signal is similar to that of thermal noise.

Expression for Quantization Noise and SNR in PCM:-

Let Q = Random Variable denotes the Quantization error
q = Sampled value of Q

Assuming that the random variable Q is uniformly distributed over the possible range (-Δ/2 to Δ/2) , as

$$f_Q(q) = \begin{cases} 1/\Delta & -\Delta/2 \le q \le \Delta/2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{------- } (3.3)$$

where $f_Q(q)$ = probability density function of the Quantization error. If the signal does not overload the Quantizer, then the mean of Quantization error is zero and its variance σ_Q^2 .

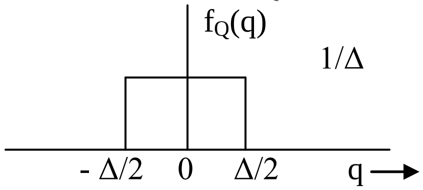

Fig:3.8 PDF for Quantization error.

Therefore

$$\sigma_Q^2 = E\{Q^2\}$$

$$\sigma_Q^2 = \int_{-\infty}^{\infty} q^2 f_q(q) dq \qquad \text{---- (3.4)}$$

$$\sigma_Q^2 = \frac{1}{\Delta}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2 dq = \frac{\Delta^2}{12} \qquad \text{--- (3.5)}$$

Thus the variance of the Quantization noise produced by a Uniform Quantizer, grows as the square of the step size. Equation (3.5) gives an expression for Quantization noise in PCM system.

Let σ_X^2 = Variance of the base band signal x(t) at the input of Quantizer.

When the base band signal is reconstructed at the receiver output, we obtain original signal plus Quantization noise. Therefore output signal to Quantization noise ration (SNR) is given by

$$(SNR)_O = \frac{Signal\ Power}{Noise\ Power} = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{\sigma_X^2}{\Delta^2/12} \qquad \text{---------- (3.6)}$$

Smaller the step size Δ, larger will be the SNR.

Signal to Quantization Noise Ratio:- [Mid Tread Type]

Let x = Quantizer input, sampled value of random variable X with mean X, variance σ_X^2. The Quantizer is assumed to be uniform, symmetric and mid tread type.

x_{max} = absolute value of the overload level of the Quantizer.

Δ = Step size

L = No. of Quantization level given by

$$L = \frac{2x_{\max}}{\Delta} + 1 \qquad \text{----- (3.7)}$$

Let n = No. of bits used to represent each level.
In general $2^n = L$, but in the mid tread Quantizer, since the number of representation levels is odd,

$L = 2^n - 1$ --------- (Mid tread only) ---- (3.8)

From the equations 3.7 and 3.8,

$$2^n - 1 = \frac{2x_{\max}}{\Delta} + 1$$

Or

$$\Delta = \frac{x_{\max}}{2^{n-1} - 1} \qquad \text{---- (3.9)}$$

The ratio $\frac{x_{\max}}{\sigma_x}$ is called the loading factor. To avoid significant overload distortion, the amplitude of the Quantizer input x extend from $-4\sigma_x$ to $4\sigma_x$, which corresponds to loading factor of 4. Thus with $x_{\max} = 4\sigma_x$ we can write equation (3.9) as

$$\Delta = \frac{4\sigma_x}{2^{n-1} - 1} \qquad \text{----------(3.10)}$$

$$(SNR)_O = \frac{\sigma_X{}^2}{\Delta^2/12} = \frac{3}{4}[2^{n-1} - 1]^2 \qquad \text{-------------(3.11)}$$

For larger value of n (typically n>6), we may approximate the result as

$$(SNR)_O = \frac{3}{4}[2^{n-1} - 1]^2 \approx \frac{3}{16}(2^{2n}) \qquad \text{--------------- (3.12)}$$

Hence expressing SNR in db

$$10 \log_{10} (SNR)_O = 6n - 7.2 \qquad \text{--------------- (3.13)}$$

This formula states that each bit in codeword of a PCM system contributes 6db to the signal to noise ratio.

For loading factor of 4, the problem of overload i.e. the problem that the sampled value of signal falls outside the total amplitude range of Quantizer, $8\sigma_x$ is less than 10^{-4}.

The equation 3.11 gives a good description of the noise performance of a PCM system provided that the following conditions are satisfied.

1. The Quantization error is uniformly distributed

2. The system operates with an average signal power above the error threshold so that the effect of channel noise is made negligible and performance is there by limited essentially by Quantization noise alone.
3. The Quantization is fine enough (say n>6) to prevent signal correlated patterns in the Quantization error waveform
4. The Quantizer is aligned with input for a loading factor of 4

Note: 1. Error uniformly distributed
2. Average signal power
3. n > 6
4. Loading factor = 4

From (3.13): $10 \log_{10} (SNR)_O = 6n - 7.2$

In a PCM system, Bandwidth B = nW or [n=B/W]
substituting the value of 'n' we get
$10 \log_{10} (SNR)_O = 6(B/W) - 7.2$ --------(3.14)

Signal to Quantization Noise Ratio:- [Mid Rise Type]

Let x = Quantizer input, sampled value of random variable X with mean X variance $\sigma_X{}^2$.
The Quantizer is assumed to be uniform, symmetric and mid rise type.
Let x_{max} = absolute value of the overload level of the Quantizer.

$$L = \frac{2x_{\max}}{\Delta} \qquad \text{------------------(3.15)}$$

Since the number of representation levels is even,

$L = 2^n$ ------- (Mid rise only) ---- (3.16)

From (3.15) and (3.16)

$$\Delta = \frac{x_{\max}}{2^n} \qquad \text{-------------- (3.17)}$$

$$(SNR)_O = \frac{\sigma_X{}^2}{\Delta^2/12} \qquad \text{-------------(3.18)}$$

where $\sigma_X{}^2$ represents the variance or the signal power.

Consider a special case of Sinusoidal signals:

Let the signal power be Ps, then Ps = 0.5 x^2_{max}.

$$(SNR)_O = \frac{Ps}{\Delta^2/12} = \frac{12\ Ps}{\Delta^2} = 1.5 L^2 = 1.5\ 2^{2n} \quad \text{-----(3.19)}$$

In decibels, $(SNR)_0$ = 1.76 + 6.02 n -----------(3.20)

Improvement of SNR can be achieved by increasing the number of bits, n. Thus for 'n' number of bits / sample the SNR is given by the above equation 3.19. For every increase of one bit / sample the step size reduces by half. Thus for (n+1) bits the SNR is given by

$(SNR)_{(n+1)\ bit} = (SNR)_{(n)\ bit} + 6dB$

Therefore addition of each bit increases the SNR by 6dB

Problem-1: An analog signal is sampled at the Nyquist rate fs = 20K and quantized into L=1024 levels. Find Bit-rate and the time duration Tb of one bit of the binary encoded signal.

Solution: Assume Mid-rise type, n = $\log_2 L$ = 10
Bit-rate = Rb = nfs = 200K bits/sec
Bit duration Tb = 1/ Rb = 5μsec.

Problem-2: A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is 56Mega bits/sec. Find the output signal-to-quantization noise ratio when a sinusoidal wave of 1MHz frequency is applied to the input.

Solution:
Given n = 7 and bit rate Rb = 56 Mega bits per second.
Sampling frequency = Rb/n = 8MHz
Message bandwidth = 4MHz.
For Mid-rise type
$(SNR)_0$ = 43.9 dB

CLASSIFICATION OF QUANTIZATION NOISE:

The Quantizing noise at the output of the PCM decoder can be categorized into four types depending on the operating conditions:

Overload noise, Random noise, Granular Noise and Hunting noise

OVER LOAD NOISE:- The level of the analog waveform at the input of the PCM encoder needs to be set so that its peak value does not exceed the design peak of Vmax volts. If the peak input does exceed Vmax, then the recovered analog waveform at the output of the PCM system will have flat – top near the peak values. This produces overload noise.

GRANULAR NOISE:- If the input level is reduced to a relatively small value w.r.t to the design level (quantization level), the error values are not same from sample to sample and the noise has a harsh sound resembling gravel being poured into a barrel. This is granular noise.

This noise can be randomized (noise power decreased) by increasing the number of quantization levels i.e.. increasing the PCM bit rate.

HUNTING NOISE:- This occurs when the input analog waveform is nearly constant. For these conditions, the sample values at the Quantizer output can oscillate between two adjacent quantization levels, causing an undesired sinusoidal type tone of frequency (0.5fs) at the output of the PCM system

This noise can be reduced by designing the quantizer so that there is no vertical step at constant value of the inputs.

ROBUST QUANTIZATION

Features of an uniform Quantizer

- Variance is valid only if the input signal does not overload Quantizer
- SNR Decreases with a decrease in the input power level.

A Quantizer whose SNR remains essentially constant for a wide range of input power levels. A quantizer that satisfies this requirement is said to be robust. The provision for such robust performance necessitates the use of a non-uniform quantizer. In a non-uniform quantizer the step size varies. For smaller amplitude ranges the step size is small and larger amplitude ranges the step size is large.

In Non – Uniform Quantizer the step size varies. The use of a non – uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. The resultant signal is then transmitted.

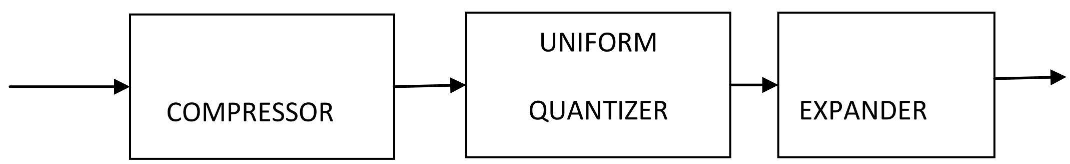

Fig: 3.9 MODEL OF NON UNIFORM QUANTIZER

At the receiver, a device with a characteristic complementary to the compressor called Expander is used to restore the signal samples to their correct relative level.

The Compressor and expander take together constitute a Compander.

Compander = Compressor + Expander

Advantages of Non – Uniform Quantization :

1. Higher average signal to quantization noise power ratio than the uniform quantizer when the signal pdf is non uniform which is the case in many practical situation.
2. RMS value of the quantizer noise power of a non – uniform quantizer is substantially proportional to the sampled value and hence the effect of the quantizer noise is reduced.

Expression for quantization error in non-uniform quantizer:

The Transfer Characteristics of the compressor and expander are denoted by C(x) and $C^{-1}(x)$ respectively, which are related by,

$$C(x).\, C^{-1}(x) = 1 \qquad \text{------ (3.21)}$$

The Compressor Characteristics for large L and x inside the interval I_k:

$$\frac{dc(x)}{dx} = \frac{2x_{\max}}{L\Delta_k} \; for \; k = 0,1,.....L-1 \qquad \text{---------- (3.22)}$$

where Δ_k = Width in the interval $I_{k.}$

Let $f_X(x)$ be the PDF of 'X' .

Consider the two assumptions:

- $f_X(x)$ is Symmetric
- $f_X(x)$ is approximately constant in each interval. ie.. $f_X(x) = f_X(y_k)$

$\Delta k = x_{k+1} - x_k$ for k = 0, 1, … L-1. ------(3.23)

Let p_k = Probability of variable X lies in the interval $I_{k,}$ then

$p_k = P(x_k < X \leq x_{k+1}) = f_X(x)\, \Delta k = f_X(y_k)\, \Delta k$ ------ (3.24)

with the constraint $\displaystyle\sum_{k=0}^{L-1} p_k = 1$

Let the random variable Q denote the quantization error, then

Q = yk – X for xk < X $\leq$ xk+1

Variance of Q is

$\sigma_Q^2 = E(Q^2) = E[(X - y_k)^2]$ ---- (3.25)

$$\sigma_Q{}^2 = \int_{-x_{\max}}^{+x_{\max}} (x - y_k)^2 \; f_X(x)\; dx \qquad \text{---- (3.26)}$$

Dividing the region of integration into L intervals and using (3.24)

$$\sigma_Q{}^2 = \sum_{k=0}^{L-1} \frac{p_k}{\Delta_k} \int_{x_k}^{x_{k+1}} (x - y_k)^2\, dx \qquad \text{----- (3.27)}$$

Using $y_k = 0.5\,(x_k + x_{k+1})$ in 3.27 and carrying out the integration w.r.t x, we obtain that

$$\sigma_Q^{\ 2} = \frac{1}{12}\sum_{k=0}^{L-1} p_k\, \Delta_k^2 \qquad \text{------- (3.28)}$$

Compression Laws.

Two Commonly used logarithmic compression laws are called μ - law and A – law.

μ-law:

In this companding, the compressor characteristics is defined by equation 3.29. The normalized form of compressor characteristics is shown in the figure 3.10. The μ-law is used for PCM telephone systems in the USA, Canada and Japan. A practical value for μ is 255.

$$\frac{c(|x|)}{x_{\max}} = \frac{\ln(1+\mu|x|/x_{\max})}{\ln(1+\mu)} \qquad 0 \le \frac{|x|}{x_{\max}} \le 1 \qquad \text{----(3.29)}$$

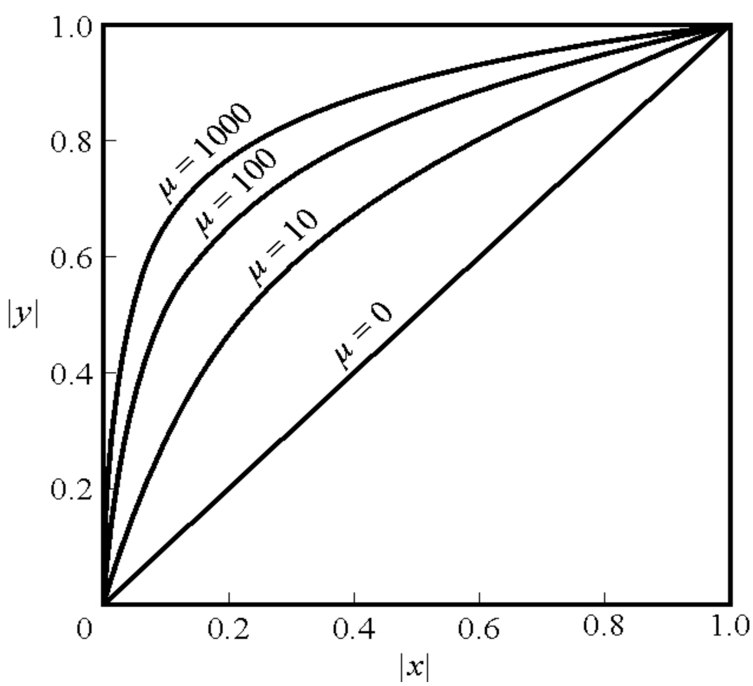

Fig: 3.10 Compression characteristics of μ-law

A-law:

In A-law companding the compressor characteristics is defined by equation 3.30. The normalized form of A-law compressor characteristics is shown in the figure 3.11. The A-law is used for PCM telephone systems in Europe. A practical value for A is 100.

$$\frac{c(|x|)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\ln A} & 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln\left(A|x|/x_{max}\right)}{1+\ln A} & \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$

------------- (3.30)

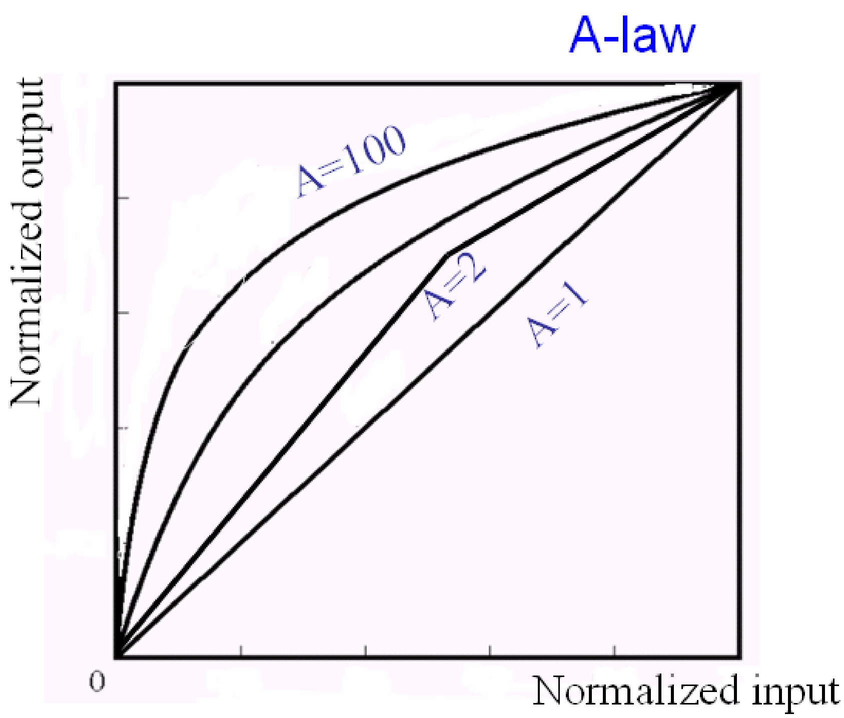

Fig. 3.11: A-law compression Characteristics.

Advantages of Non Uniform Quantizer

- Reduced Quantization noise
- High average SNR

Differential Pulse Code Modulation (DPCM)

For the signals which does not change rapidly from one sample to next sample, the PCM scheme is not preferred. When such highly correlated samples are encoded the resulting encoded signal contains redundant information. By removing this redundancy before encoding an efficient coded signal can be obtained. One of such scheme is the DPCM technique. By knowing the past behavior of a signal up to a certain point in time, it is possible to make some inference about the future values.

The transmitter and receiver of the DPCM scheme is shown in the fig3.12 and fig 3.13 respectively.

Transmitter: Let x(t) be the signal to be sampled and x(nTs) be it's samples. In this scheme the input to the quantizer is a signal

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \qquad \text{-----} \ (3.31)$$

where x^(nTs) is the prediction for unquantized sample x(nTs). This predicted value is produced by using a predictor whose input, consists of a quantized versions of the input signal x(nTs). The signal e(nTs) is called the prediction error.

By encoding the quantizer output, in this method, we obtain a modified version of the PCM called differential pulse code modulation (DPCM).

Quantizer output, $v(nT_s) = Q[e(nT_s)]$

$$= e(nT_s) + q(nT_s) \qquad \text{----} \ (3.32)$$

where q(nTs) is the quantization error.

Predictor input is the sum of quantizer output and predictor output,

$$u(nT_s) = \hat{x}(nT_s) + v(nT_s) \qquad \text{----} \ (3.33)$$

Using 3.32 in 3.33, $u(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$ ----(3.34)

$$u(nT_s) = x(nT_s) + q(nT_s) \qquad \text{----}(3.35)$$

The receiver consists of a decoder to reconstruct the quantized error signal. The quantized version of the original input is reconstructed from the decoder output using the same predictor as used in the transmitter. In the absence of noise the encoded signal at the receiver input is identical to the encoded signal at the transmitter output. Correspondingly the receive output is equal to u(nTs), which differs from the input x(nts) only by the quantizing error q(nTs).

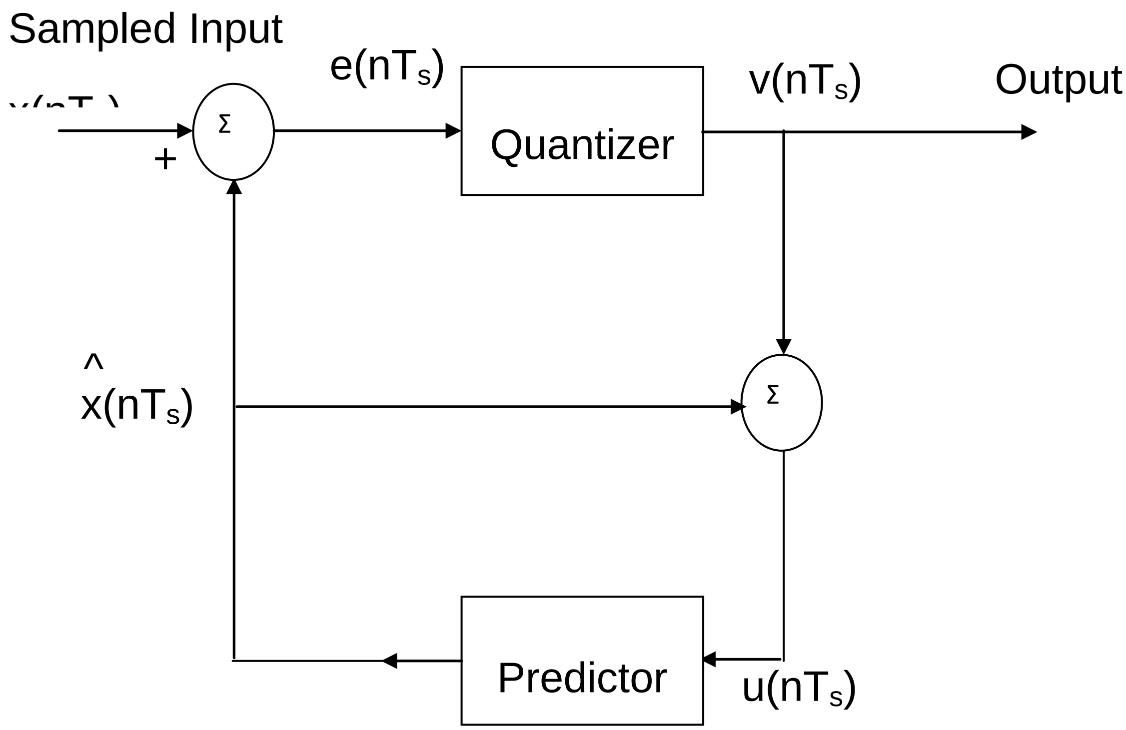

Fig:3.12 - Block diagram of DPCM Transmitter

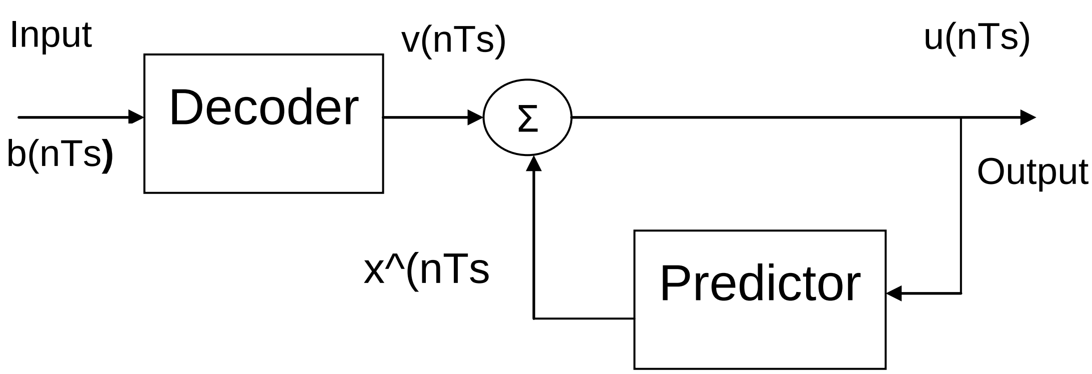

Fig:3.13 - Block diagram of DPCM Receiver.

Prediction Gain (Gp):

The output signal-to-quantization noise ratio of a signal coder is defined as

$$(SNR)_0 = \frac{\sigma_X^2}{\sigma_Q^2} \quad \text{--------------------(3.36)}$$

where σ_x^2 is the variance of the signal x(nTs) and σ_Q^2 is the variance of the quantization error q(nTs). Then

$$(SNR)_0 = \left(\frac{\sigma_X^2}{\sigma_E^2}\right)\left(\frac{\sigma_E^2}{\sigma_Q^2}\right) = G_P\,(SNR)_P \quad \text{------(3.37)}$$

where σ_E^2 is the variance of the prediction error e(nTs) and $(SNR)_P$ is the prediction error-to-quantization noise ratio, defined by

$$(SNR)_P = \frac{\sigma_E^2}{\sigma_Q^2} \quad \text{--------------(3.38)}$$

The Prediction gain Gp is defined as

$$G_P = \frac{\sigma_X^2}{\sigma_E^2} \quad \text{--------(3.39)}$$

The prediction gain is maximized by minimizing the variance of the prediction error. Hence the main objective of the predictor design is to minimize the variance of the prediction error.

The prediction gain is defined by $G_P = \dfrac{1}{(1-\rho_1^2)}$ ---- (3.40)

and $\sigma_E^2 = \sigma_X^2\,(1-\rho_1^2)$ ----(3.41)

where ρ_1 – Autocorrelation function of the message signal

PROBLEM:

Consider a DPCM system whose transmitter uses a first-order predictor optimized in the minimum mean-square sense. Calculate the prediction gain of the system for the following values of correlation coefficient for the message signal:

$$(i)\,\rho_1 = \frac{R_x(1)}{R_x(0)} = 0.825 \qquad (ii)\,\rho_1 = \frac{R_x(1)}{R_x(0)} = 0.950$$

Solution:

Using (3.40)

(i) For ρ_1= 0.825, Gp = 3.13 In dB , Gp = 5dB

(ii) For ρ_2 = 0.95, Gp = 10.26 In dB, Gp = 10.1dB

Delta Modulation (DM)

Delta Modulation is a special case of DPCM. In DPCM scheme if the base band signal is sampled at a rate much higher than the Nyquist rate purposely to increase the correlation between adjacent samples of the signal, so as to permit the use of a simple quantizing strategy for constructing the encoded signal, Delta modulation (DM) is precisely such as scheme. Delta Modulation is the one-bit (or two-level) versions of DPCM.

DM provides a staircase approximation to the over sampled version of an input base band signal. The difference between the input and the approximation is quantized into only two levels, namely, $\pm\delta$ corresponding to positive and negative differences, respectively, Thus, if the approximation falls below the signal at any sampling epoch, it is increased by δ. Provided that the signal does not change too rapidly from sample to sample, we find that the stair case approximation remains within $\pm\delta$ of the input signal. The symbol δ denotes the absolute value of the two representation levels of the one-bit quantizer used in the DM. These two levels are indicated in the transfer characteristic of Fig 3.14. The step size Δ of the quantizer is related to δ by

$$\Delta = 2\delta \qquad \text{-----} \quad (3.42)$$

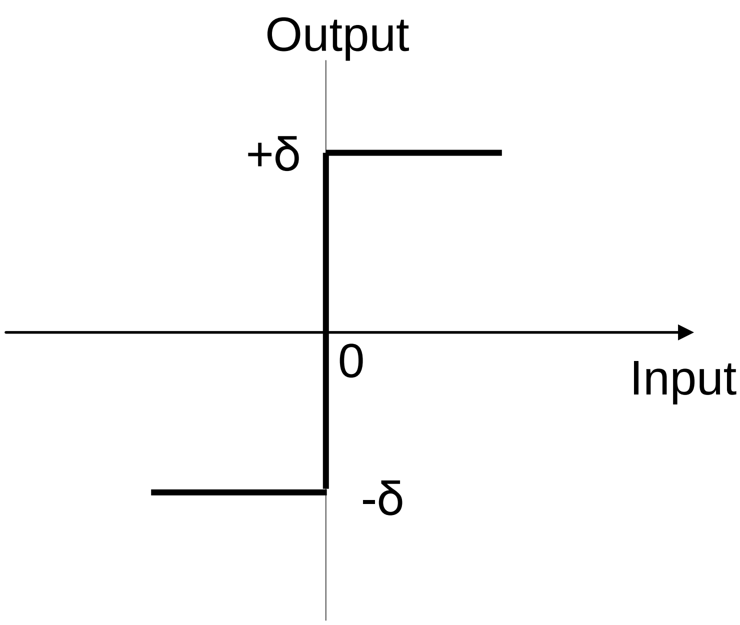

Fig-3.14: Input-Output characteristics of the delta modulator.

Let the input signal be x(t) and the staircase approximation to it is u(t). Then, the basic principle of delta modulation may be formalized in the following set of relations:

$$e(nT_s) = x(nT_s) - x^{\wedge}(nT_s)$$
$$e(nTs) = x(nTs) - u(nTs - Ts)$$
$$b(nT_s) = \delta \ \mathrm{sgn}[e(nT_s)] \quad and \qquad \text{----- (3.43)}$$
$$u(nT_s) = u(nT_s - T_s) + b(nT_s)$$

where T_s is the sampling period; $e(nT_s)$ is a prediction error representing the difference between the present sample value $x(nT_s)$ of the input signal and the latest approximation to it, namely $\hat{x}(nT_s) = u(nT_s - T_s)$ .The binary quantity, $b(nT_s)$ is the one-bit word transmitted by the DM system.

The transmitter of DM system is shown in the figure3.15. It consists of a summer, a two-level quantizer, and an accumulator. Then, from the equations of (3.43) we obtain the output as,

$$u(nTs) = \delta \sum_{i=1}^{n} \mathrm{sgn}[e(iTs)] = \sum_{i=1}^{n} b(iTs) \qquad \text{----- (3.44)}$$

At each sampling instant, the accumulator increments the approximation to the input signal by ±δ, depending on the binary output of the modulator.

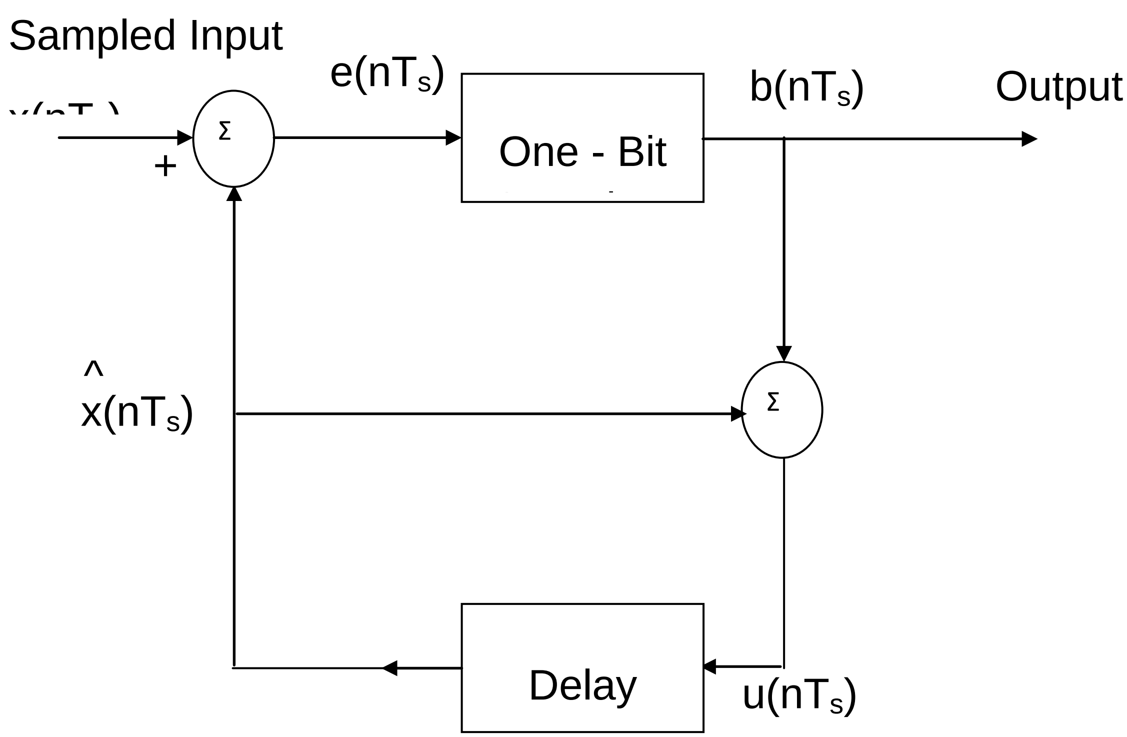

Fig 3.15 - Block diagram for Transmitter of a DM system

In the receiver, shown in fig.3.16, the stair case approximation u(t) is reconstructed by passing the incoming sequence of positive and negative pulses through an accumulator in a manner similar to that used in the transmitter. The out-of –band quantization noise in the high frequency staircase waveform u(t) is rejected by passing it through a low-pass filter with a band-width equal to the original signal bandwidth.

Delta modulation offers two unique features:

1. No need for Word Framing because of one-bit code word.
2. Simple design for both Transmitter and Receiver

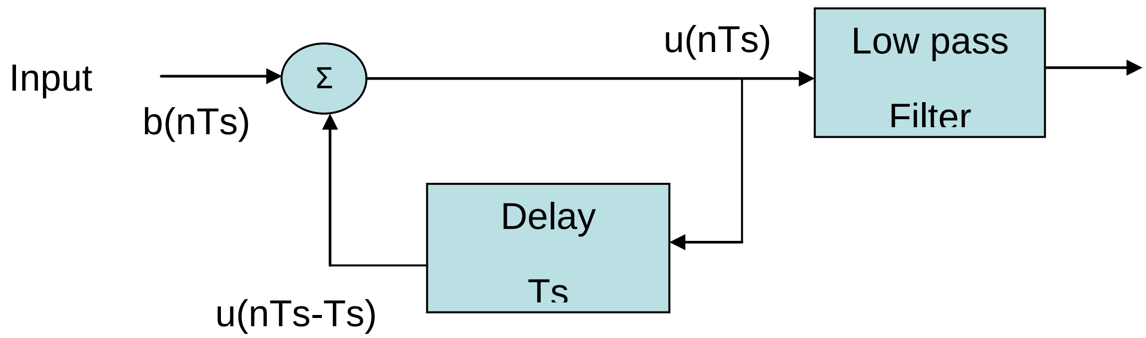

Fig 3.16 - Block diagram for Receiver of a DM system

QUANTIZATION NOISE

Delta modulation systems are subject to two types of quantization error:
(1) slope –overload distortion, and (2) granular noise.

If we consider the maximum slope of the original input waveform x(t), it is clear that in order for the sequence of samples$\{u(nT_s)\}$ to increase as fast as the input sequence of samples $\{x(nT_s)\}$ in a region of maximum slope of x(t), we require that the condition in equation 3.45 be satisfied.

$$\frac{\delta}{T_s} \geq \max\left|\frac{dx(t)}{dt}\right| \qquad \text{------- (3.45)}$$

Otherwise, we find that the step size $\Delta = 2\delta$ is too small for the stair case approximation u(t) to follow a steep segment of the input waveform x(t), with the result that u(t) falls behind x(t). This condition is called slope-overload, and the resulting quantization error is called slope-overload distortion(noise). Since the maximum slope of the staircase approximation u(t) is fixed by the step size Δ, increases and decreases in u(t) tend to occur along straight lines. For this reason, a delta modulator using a fixed step size is often referred ton as linear delta modulation (LDM).

The granular noise occurs when the step size Δ is too large relative to the local slope characteristics of the input wave form x(t), thereby causing the staircase approximation u(t) to hunt around a relatively flat segment of the input waveform; The granular noise is analogous to quantization noise in a PCM system.

The e choice of the optimum step size that minimizes the mean-square value of the quantizing error in a linear delta modulator will be the result of a compromise between slope overload distortion and granular noise.

Output SNR for Sinusoidal Modulation.

Consider the sinusoidal signal, x(t) = A cos(2μfot)
The maximum slope of the signal x(t) is given by

$$\max\left|\frac{dx(t)}{dt}\right| = 2\pi f_0 A \qquad \text{----- (3.46)}$$

The use of Eq.5.81 constrains the choice of step size Δ = 2δ, so as to avoid slope-overload. In particular, it imposes the following condition on the value of δ:

$$\frac{\delta}{T_s} \geq \max\left|\frac{dx(t)}{dt}\right| = 2\pi f_0 A \qquad \text{----- (3. 47)}$$

Hence for no slope overload error the condition is given by equations 3.48 and 3.49.

$$A \leq \frac{\delta}{2\pi f_0 Ts} \qquad \text{------ (3.48)}$$

$$\delta \geq 2\pi f_0 A T_s \qquad \text{------ (3.49)}$$

Hence, the maximum permissible value of the output signal power equals

$$P_{\max} = \frac{A^2}{2} = \frac{\delta^2}{8\pi^2 f_0^2 T_s^2} \qquad \text{---- (3.50)}$$

When there is no slope-overload, the maximum quantization error ±δ. Assuming that the quantizing error is uniformly distributed (which is a reasonable approximation for small δ). Considering the probability density function of the quantization error,(defined in equation 3.51),

$$f_Q(q) = \frac{1}{2\delta} \;\; for \;\; -\delta \le \; q \le +\delta$$
$$0 \quad otherwise \qquad \text{----- (3.51)}$$

The variance of the quantization error is $\sigma^2{}_Q$.

$$\sigma_Q{}^2 = \frac{1}{2\delta} \int_{-\delta}^{+\delta} q^2 \;\; dq = \frac{\delta^2}{3} \qquad \text{----- (3.52)}$$

The receiver contains (at its output end) a low-pass filter whose bandwidth is set equal to the message bandwidth (i.e., highest possible frequency component of the message signal), denoted as W such that $f_0 \le$ W. Assuming that the average power of the quantization error is uniformly distributed over a frequency interval extending from $-1/T_s$ to $1/T_s$, we get the result:

Average output noise power $N_o = \left(\frac{f_c}{f_s}\right)\frac{\delta^2}{3} = WT_s\left(\frac{\delta^2}{3}\right)$ ----- (3.53)

Correspondingly, the maximum value of the output signal-to-noise ratio equals

$$(SNR)_O = \frac{P_{\max}}{N_o} = \frac{3}{8\pi^2 W f_0^2 T_s^3} \qquad \text{----- (3.54)}$$

Equation 3.54 shows that, under the assumption of no slope-overload distortion, the maximum output signal-to-noise ratio of a delta modulator is proportional to the sampling rate cubed. This indicates a 9db improvement with doubling of the sampling rate.

Problems

1. **Determine the output SNR in a DM system for a 1KHz sinusoid sampled at 32KHz without slope overload and followed by a 4KHz post reconstruction filter.**

Solution:

Given W=4KHz, f0 = 1KHz , fs = 32KHz
Using equation (3.54) we get

$(SNR)_0$ = 311.3 or 24.9dB

__Delta Modulation:__

__Problems__

2. Consider a Speech Signal with maximum frequency of 3.4KHz and maximum amplitude of 1volt. This speech signal is applied to a delta modulator whose bit rate is set at 60kbit/sec. Explain the choice of an appropriate step size for the modulator.

Solution: Bandwidth of the signal = 3.4 KHz.
Maximum amplitude = 1 volt
Bit Rate = 60Kbits/sec
Sampling rate = 60K Samples/sec.
STEP SIZE = 0.356 Volts

3. Consider a Speech Signal with maximum frequency of 3.4KHz and maximum amplitude of 1volt. This speech signal is applied to a delta modulator whose bit rate is set at 20kbit/sec. Explain the choice of an appropriate step size for the modulator.

Solution: Bandwidth of the signal = 3.4 KHz.
Maximum amplitude = 1 volt
Bit Rate = 20Kbits/sec
Sampling rate = 20K Samples/sec.
STEP SIZE = 1.068 Volts

4. Consider a Delta modulator system designed to operate at 4 times the Nyquist rate for a signal with a 4KHz bandwidth. The step size of the quantizer is 400mV.
a) Find the maximum amplitude of a 1KHz input sinusoid for which the delta modulator does not show slope overload.
b) Find post-filtered output SNR

Solution: Bandwidth of the signal = f0 =1 KHz.
Nyquist Rate = 8K samples/sec
Sampling Rate = 32K samples/sec.
Step Size = 400 mV

a) For 1KHz sinusoid, Amax = 2.037 volts.
b) Assuming LPF bandwidth = W= 4KHz
   SNR = 311.2586 = 24.93 dB

__Adaptive Delta Modulation:__

The performance of a delta modulator can be improved significantly by making the step size of the modulator assume a time-varying form. In particular, during a steep segment of the input signal the step size is increased. Conversely, when the input signal is varying slowly, the step size is reduced. In this way, the size is adapted to the level of the input signal. The resulting method is called adaptive delta modulation (ADM).

There are several types of ADM, depending on the type of scheme used for adjusting the step size. In this ADM, a discrete set of values is provided for the step size. Fig.3.17 shows the block diagram of the transmitter and receiver of an ADM System.

In practical implementations of the system, the step size

$$\Delta(nT_s) \text{ or } 2\delta(nT_s)$$

is constrained to lie between minimum and maximum values.

The upper limit, $\delta_{\max}$, controls the amount of slope-overload distortion. The lower limit, $\delta_{\min}$, controls the amount of idle channel noise. Inside these limits, the adaptation rule for $\delta(nT_s)$ is expressed in the general form

$$\delta(nTs) = g(nTs).\ \delta(nTs - Ts) \qquad \text{------ (3.55)}$$

where the time-varying multiplier $g(nT_s)$ depends on the present binary output $b(nT_s)$ of the delta modulator and the M previous values $b(nT_s - T_s), \ldots\ldots. b(nT_s - MT_s)$.

This adaptation algorithm is called a constant factor ADM with one-bit memory, where the term “one bit memory” refers to the explicit utilization of the single pervious bit $b(nT_s - T_s)$ because equation (3.55) can be written as,

$$g(nTs) = K \quad \text{if } b(nTs) = b(nTs - Ts)$$
$$g(nTs) = K^{-1} \quad \text{if } b(nTs) = b(nTs - Ts) \qquad \text{------ (3.56)}$$

This algorithm of equation (3.56), with K=1.5 has been found to be well matched to typically speech and image inputs alike, for a wide range of bit rates.

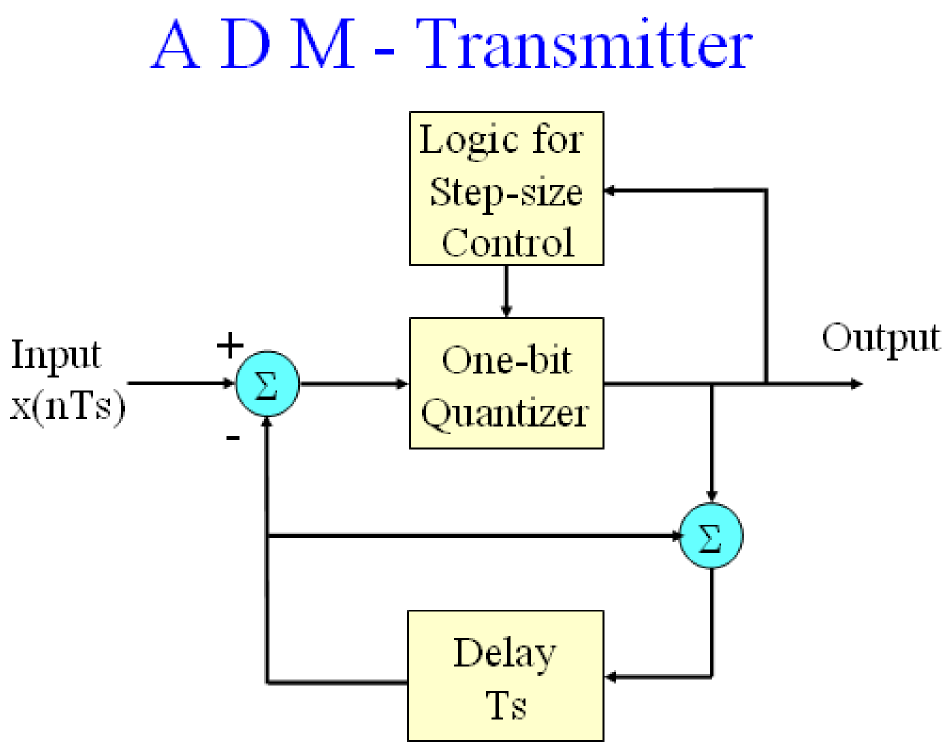

Figure: 3.17a) Block Diagram of ADM Transmitter.

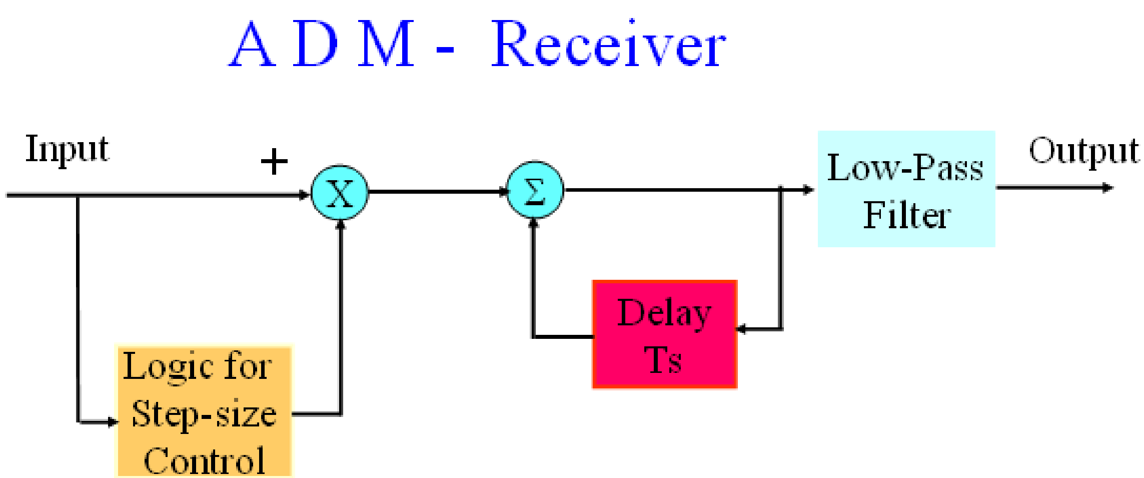

Figure: 3.17 b): Block Diagram of ADM Receiver.

Coding Speech at Low Bit Rates:

The use of PCM at the standard rate of 64 kb/s demands a high channel bandwidth for its transmission. But channel bandwidth is at a premium, in which case there is a definite need for speech coding at low bit rates, while maintaining acceptable fidelity or quality of reproduction. The fundamental limits on bit rate suggested by speech perception and information theory show that high quality speech coding is possible at rates considerably less that 64 kb/s (the rate may actually be as low as 2 kb/s).

For coding speech at low bit rates, a waveform coder of prescribed configuration is optimized by exploiting both statistical characterization of speech waveforms and properties of hearing. The design philosophy has two aims in mind:

1. To remove redundancies from the speech signal as far as possible.
2. To assign the available bits to code the non-redundant parts of the speech signal in a perceptually efficient manner.

To reduce the bit rate from 64 kb/s (used in standard PCM) to 32, 16, 8 and 4 kb/s, the algorithms for redundancy removal and bit assignment become increasingly more sophisticated.

There are two schemes for coding speech:

1. Adaptive Differential Pulse code Modulation (ADPCM) --- 32 kb/s
2. Adaptive Sub-band Coding.--- 16 kb/s

1. Adaptive Differential Pulse – Code Modulation

A digital coding scheme that uses both adaptive quantization and adaptive prediction is called adaptive differential pulse code modulation (ADPCM).

The term “adaptive” means being responsive to changing level and spectrum of the input speech signal. The variation of performance with speakers and speech material, together with variations in signal level inherent in the speech communication process, make the combined use of adaptive quantization and adaptive prediction necessary to achieve best performance.

The term “adaptive quantization” refers to a quantizer that operates with a time-varying step size $\Delta(nT_s)$, where T_s is the sampling period. The step size $\Delta(nT_s)$ is varied so as to match the variance $\sigma^2 x$ of the input signal $x(nT_s)$. In particular, we write

$$\Delta(nTs) = \Phi . \hat{\sigma}_x(nTs) \qquad \text{----- (3.57)}$$

where Φ – Constant

$\hat{\sigma}_x(nTs)$ – estimate of the $\sigma_x(nTs)$

Thus the problem of adaptive quantization, according to (3.57) is one of estimating $\sigma_x(nT_s)$ continuously.

The computation of the estimate $\hat{\sigma}_x(nT_s)$ in done by one of two ways:

1. Unquantized samples of the input signal are used to derive forward estimates of $\sigma_x(nT_s)$ - adaptive quantization with forward estimation (AQF)
2. Samples of the quantizer output are used to derive backward estimates of $\sigma_x(nT_s)$ - adaptive quantization with backward estimation (AQB)

The use of adaptive prediction in ADPCM is required because speech signals are inherently nonstationary, a phenomenon that manifests itself in the fact that autocorrection function and power spectral density of speech signals are time-varying functions of their respective variables. This implies that the design of predictors for such inputs should likewise be time-varying, that is, adaptive. As with adaptive quantization, there are two schemes for performing adaptive prediction:

1. Adaptive prediction with forward estimation (APF), in which unquantized samples of the input signal are used to derive estimates of the predictor coefficients.
2. Adaptive prediction with backward estimation (APB), in which samples of the quantizer output and the prediction error are used to derive estimates of the prediction error are used to derive estimates of the predictor coefficients.

(2) Adaptive Sub-band Coding:

PCM and ADPCM are both time-domain coders in that the speech signal is processed in the time-domain as a single full band signal. Adaptive sub-band coding is a frequency domain coder, in which the speech signal is divided into a number of sub-bands and each one is encoded separately. The coder is capable of digitizing speech at a rate of 16 kb/s with a quality comparable to that of 64 kb/s PCM. To accomplish this performance, it exploits the quasi-periodic nature of voiced speech and a characteristic of the hearing mechanism known as noise masking.

Periodicity of voiced speech manifests itself in the fact that people speak with a characteristic pitch frequency. This periodicity permits pitch prediction, and therefore a further reduction in the level of the prediction error that requires quantization, compared to differential pulse code modulation without pitch prediction. The number of bits per sample that needs to be transmitted is thereby greatly reduced, without a serious degradation in speech quality.

In adaptive sub band coding (ASBC), noise shaping is accomplished by adaptive bit assignment. In particular, the number of bits used to encode each sub-band is varied dynamically and shared with other sub-bands, such that the encoding accuracy is always placed where it is needed in the frequency – domain characterization of the signal. Indeed, sub-bands with little or no energy may not be encoded at all.

Applications

1. Hierarchy of Digital Multiplexers

2. Light wave Transmission Link

(1) Digital Multiplexers:

Digital Multiplexers are used to combine digitized voice and video signals as well as digital data into one data stream.

The digitized voice signals, digitized facsimile and television signals and computer outputs are of different rates but using multiplexers it combined into a single data stream.

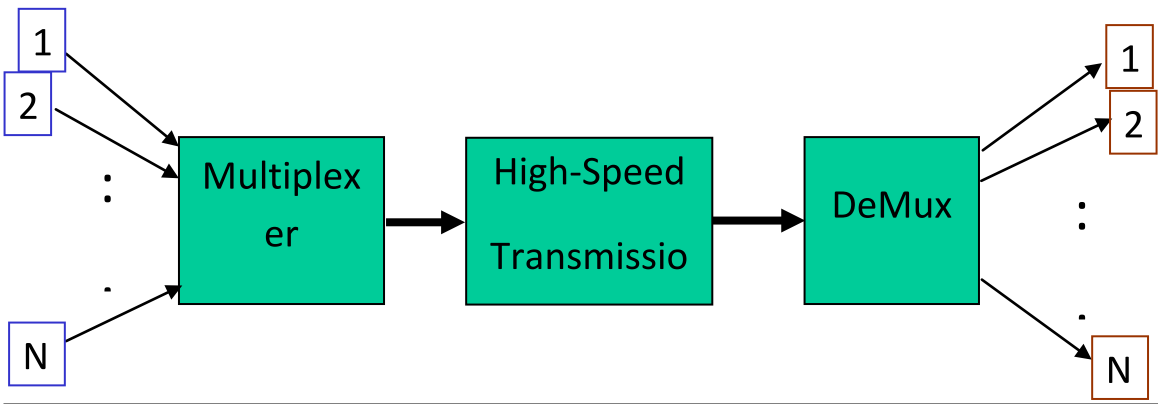

Fig. 3.18: Conceptual diagram of Multiplexing and Demultiplexing.

Two Major groups of Digital Multiplexers:

1. To combine relatively Low-Speed Digital signals used for voice-grade channels. Modems are required for the implementation of this scheme.
2. Operates at higher bit rates for communication carriers.

Basic Problems associated with Multiplexers:

1. Synchronization.
2. Multiplexed signal should include Framing.
3. Multiplexer Should be capable handling Small variations

Digital Hierarchy based on T1 carrier:

This was developed by Bell system. The T1 carrier is designed to operate at 1.544 mega bits per second, the T2 at 6.312 megabits per second, the T3 at 44.736 megabits per second, and the T4 at 274.176 mega bits per second. This system is made up of various combinations of lower order

T-carrier subsystems. This system is designed to accommodate the transmission of voice signals, Picture phone service and television signals by using PCM and digital signals from data terminal equipment. The structure is shown in the figure 3.19.

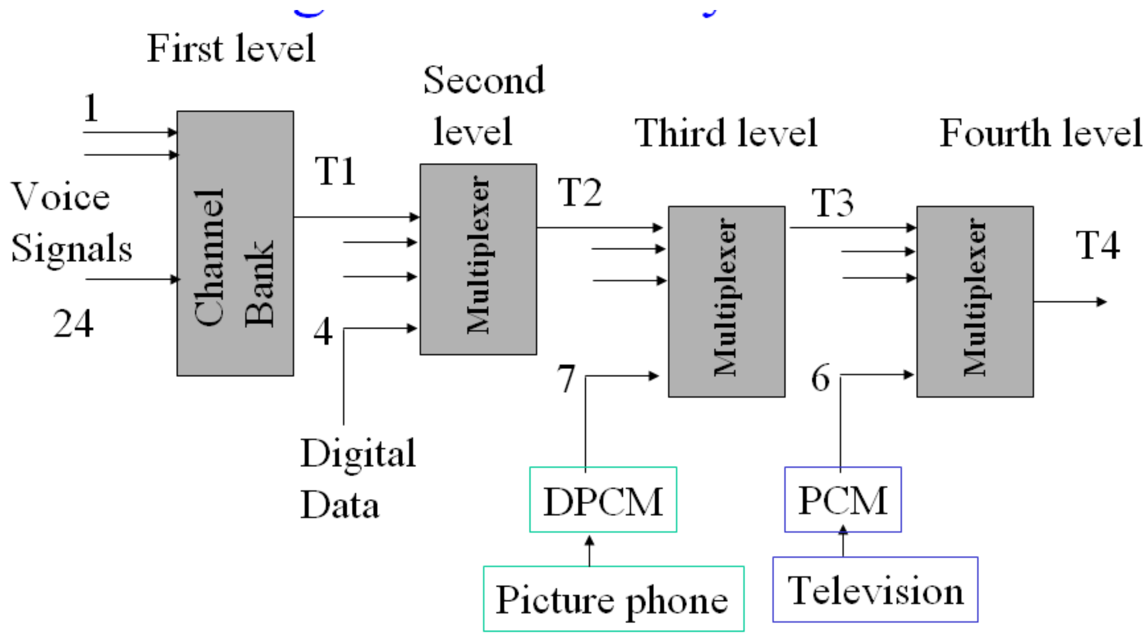

Fig. 3.19: Digital hierarchy of a 24 channel system.

The T1 carrier system has been adopted in USA, Canada and Japan. It is designed to accommodate 24 voice signals. The voice signals are filtered with low pass filter having cutoff of 3400 Hz. The filtered signals are sampled at 8KHz. The μ-law Companding technique is used with the constant $\mu = 255$.

With the sampling rate of 8KHz, each frame of the multiplexed signal occupies a period of 125μsec. It consists of 24 8-bit words plus a single bit that is added at the end of the frame for the purpose of synchronization. Hence each frame consists of a total 193 bits. Each frame is of duration 125μsec, correspondingly, the bit rate is 1.544 mega bits per second.

Another type of practical system, that is used in Europe is 32 channel system which is shown in the figure 3.20.

Fig 3.20:   32 channel TDM system

32 channel TDM Hierarchy:

In the first level 2.048 megabits/sec is obtained by multiplexing 32 voice channels.
4 frames of **32** channels = **128** PCM channels,
Data rate = 4 x 2.048 Mbit/s = **8.192 Mbit/s,**
But due to the synchronization bits the data rate increases to 8.448Mbit/sec.

4 x 128 = **512** channels
Data rate = 4 x8.192 Mbit/s (+ signalling bits) = **34.368 Mbit/s**

(2) Light Wave Transmission

Optical fiber wave guides are very useful as transmission medium. They have a very low transmission losses and high bandwidths which is essential for high-speed communications. Other advantages include small size, light weight and immunity to electromagnetic interference.

The basic optical fiber link is shown in the figure 3.21. The binary data fed into the transmitter input, which emits the pulses of optical power., with each pulse being on or off in accordance with the input data. The choice of the light source determines the optical signal power available for transmission.

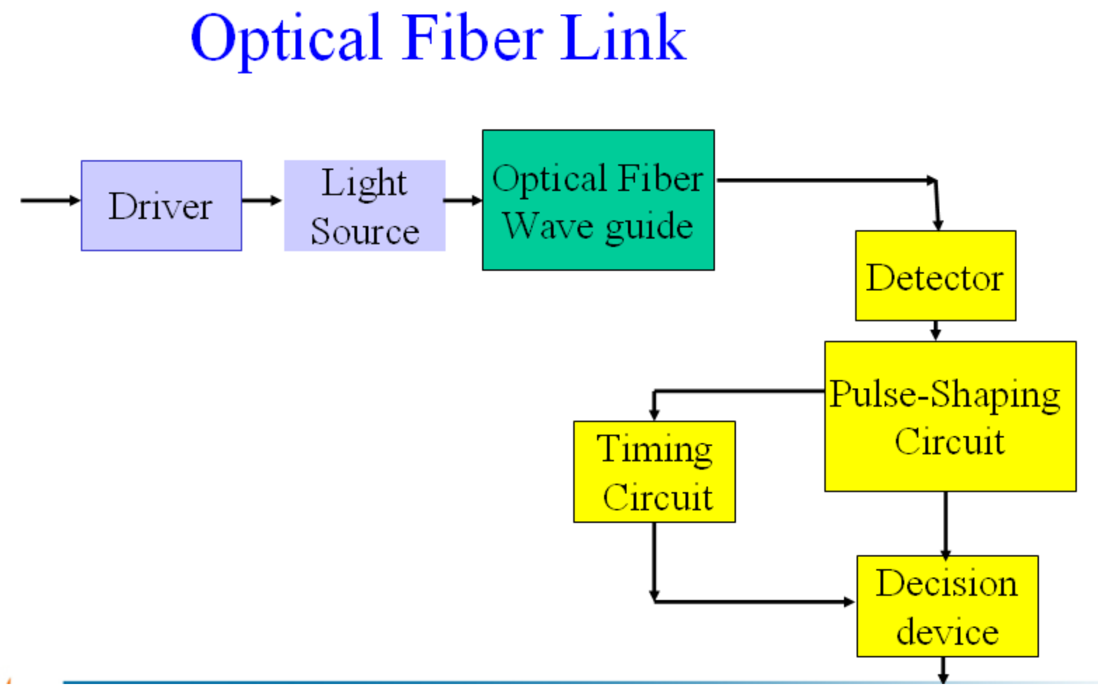

Fig: 3.21- Optical fiber link.

The on-off light pulses produced by the transmitter are launched into the optical fiber wave guide. During the course of the propagation the light pulse suffers loss or attenuation that increases exponentially with the distance.

At the receiver the original input data are regenerated by performing three basic operations which are :

1. **Detection –** the light pulses are converted back into pulses of electrical current.
2. **Pulse Shaping and Timing** - This involves amplification, filtering and equalization of the electrical pulses, as well as the extraction of timing information.
3. **Decision Making**: Depending the pulse received it should be decided that the received pulse is on or off.