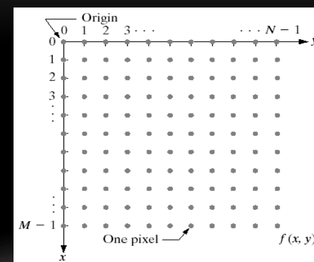
UNIT-2

- In the following slides we will consider what is involved in capturing a digital image of a real-world scene
  - Image sensing and representation
  - Image Acquisition
  - Sampling and quantisation
  - Resolution
  - Basic relationship between pixels
  - Linear and nonlinear operations

IMAGE REPRESENTATION

- Before we discuss image acquisition recall that a digital image is composed of M rows and N columns of pixels each storing a value
- Pixel values are most often grey levels in the range 0-255(black-white)
- We will see later on that images can easily be represented as matrices



col

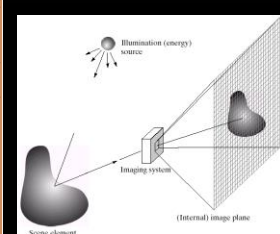
f (row, col)

row

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

IMAGE REPRESENTATION

- **Images are typically generated by *illuminating* a *scene* and absorbing the energy reflected by the objects in that scene**

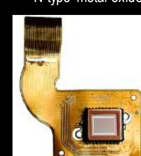


- Typical notions of illumination and scene can be way of:
  - X-rays of a skeleton
  - Ultrasound of an unborn baby
  - Electro-microscopic images of molecules

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

IMAGE SENSORS

- An image sensor is a device that converts an optical image into an electronic signal.
- It is used mostly in digital cameras, camera modules and other imaging devices.
- Early analog sensors were video camera tubes;
- Currently used types are:
  - semiconductor charge-coupled devices (CCD)
  - active pixel sensors in complementary metal–oxide–semiconductor (CMOS)
  - N-type metal-oxide-semiconductor (NMOS, Live MOS) technologies.



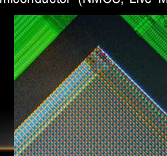


IMAGE SENSORS…

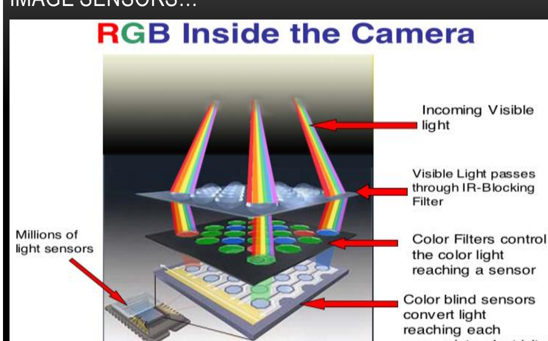
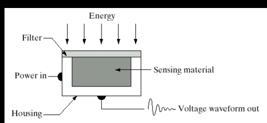


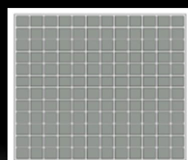
IMAGE SENSORS- FLEX CIRCUIT ASSEMBLY

IMAGE SENSING

- Incoming energy lands on a sensor material responsive to that type of energy and this generates a voltage
- There are 3 principal sensor arrangements (produce an electrical output proportional to light intensity).
- Collections of sensors are arranged to capture images
- (i) Single imaging Sensor (ii)Line sensor (iii) Array sensor



Single Imaging Sensor



Array of Image Sensors

Line of Image Sensors

IMAGE ACQUISITION USING A SINGLE SENSOR

- The most common sensor of this type is the photodiode, which is constructed of silicon materials and whose output voltage waveform is proportional to light.
- The use of a filter in front of a sensor improves selectivity.
- For example, a green (pass) filter in front of a light sensor favours light in the green band of the color spectrum.
- In order to generate a 2-D image using a single sensor, there have to be relative displacements in both the x- and y-directions between the sensor and the area to be imaged.

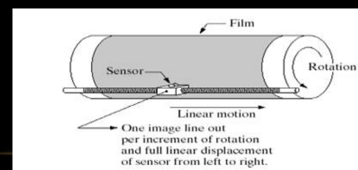


IMAGE SENSING Using Sensor Strips and Rings

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

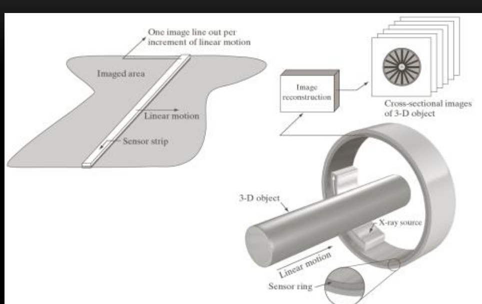


IMAGE SENSING USING SENSOR STRIPS AND RINGS

- The strip provides imaging elements in one direction.
- Motion perpendicular to the strip provides imaging in the other direction.
- This is the type of arrangement used in most flatbed scanners.
- Sensing devices with 4000 or more in-line sensors are possible.
- In-line sensors are used routinely in airborne imaging applications, in which the imaging system is mounted on an aircraft that flies at a constant altitude and speed over the geographical area to be imaged.

IMAGE SENSING USING SENSOR ARRAYS

- This type of arrangement is found in digital cameras.
- A typical sensor for these cameras is a CCD array, which can be manufactured with a broad range of sensing properties and can be packaged in rugged arrays of 4000 * 4000 elements or more.
- CCD sensors are used widely in digital cameras and other light sensing instruments.

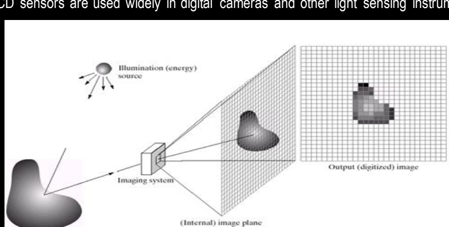


IMAGE SAMPLING AND QUANTISATION

- A digital sensor can only measure a limited number of **samples** at a **discrete** set of energy levels
- *Quantisation* is the process of converting a continuous **analogue** signal into a digital representation of this signal

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

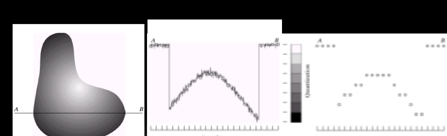


IMAGE SAMPLING AND QUANTISATION

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

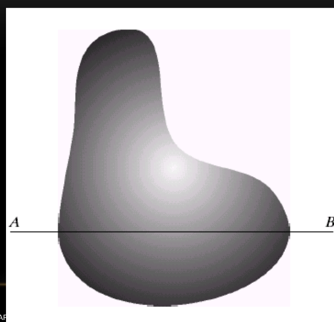
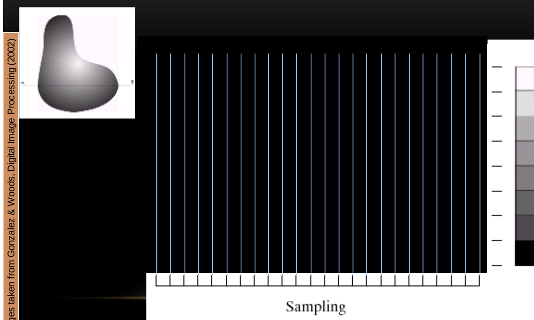


IMAGE SAMPLING AND QUANTISATION

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



A Simple Image Formation Model

- Binary images: images having only two possible brightness levels (black and white)
- Gray scale images : "black and white" images
- Color images: can be described mathematically as three gray scale images
- Let $f(x,y)$ be an image function, then

  $f(x,y) = i(x,y)\ r(x,y)$,

  where $i(x,y)$: the illumination function

  $r(x,y)$: the reflection function

  Note: $0 < i(x,y) < \infty$ and $0 < r(x,y) < 1$.
- For digital images the minimum gray level is usually 0, but the maximum depends on number of quantization levels used to digitize an image.
- The most common is 256 levels, so that the maximum level is 255.

IMAGE SAMPLING AND QUANTISATION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

- Remember that a digital image is always only an **approximation** of a real world scene

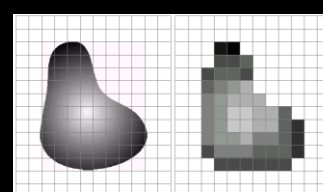


IMAGE REPRESENTATION

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

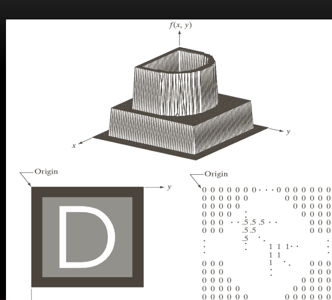


IMAGE REPRESENTATION

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

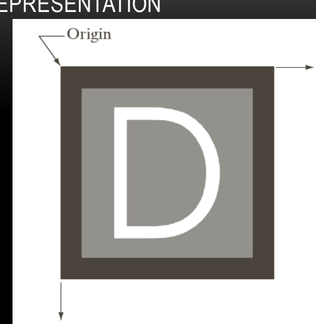


IMAGE REPRESENTATION

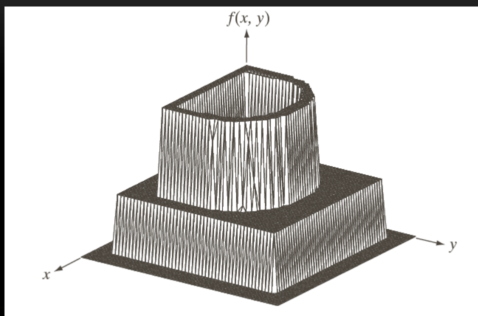
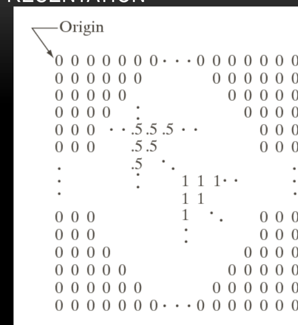


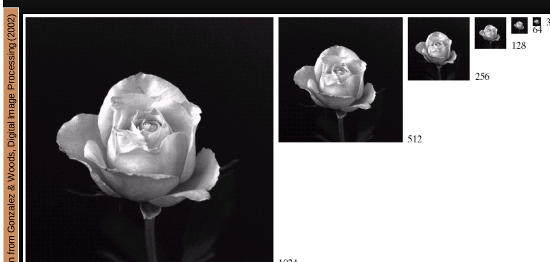
IMAGE REPRESENTATION



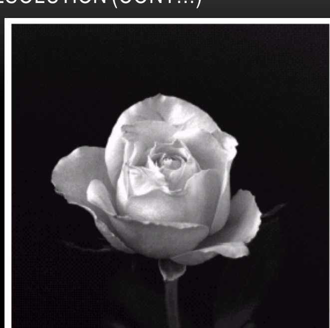
SPATIAL RESOLUTION

- *The spatial resolution* of an image is determined by how sampling was carried out
- Spatial resolution simply refers to the smallest discernable detail in an image
  - Vision specialists will often talk about pixel size
  - Graphic designers will talk about *dots per inch* (DPI)

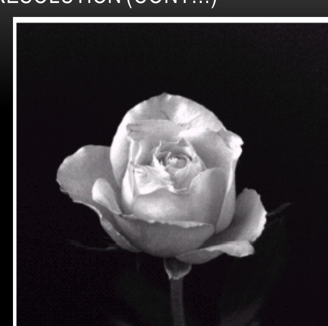
SPATIAL RESOLUTION (CONT…)



SPATIAL RESOLUTION (CONT…)



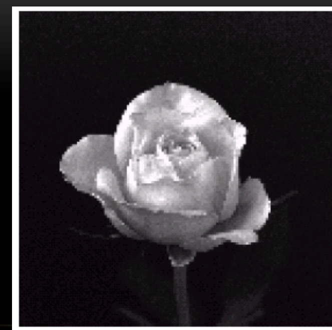
SPATIAL RESOLUTION (CONT…)



SPATIAL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

SPATIAL RESOLUTION (CONT…)



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

SPATIAL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

SPATIAL RESOLUTION (CONT…)

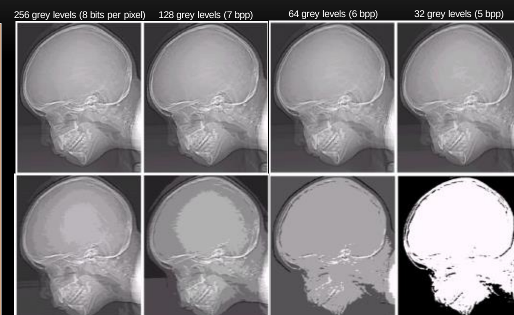
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

INTENSITY LEVEL RESOLUTION

- *Intensity level resolution* refers to the number of intensity levels used to represent the image
  - The more intensity levels used, the finer the level of detail discernable in an image
  - Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

Number of Bits	Number of Intensity Levels	Examples
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010

INTENSITY LEVEL RESOLUTION (CONT…)



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

INTENSITY LEVEL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

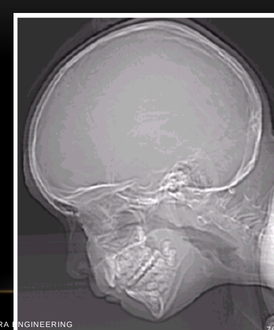
INTENSITY LEVEL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

INTENSITY LEVEL RESOLUTION (CONT…)

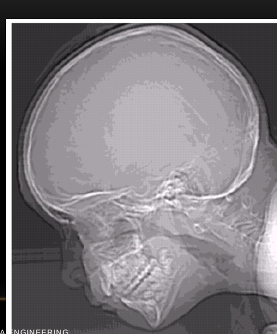
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

INTENSITY LEVEL RESOLUTION (CONT…)



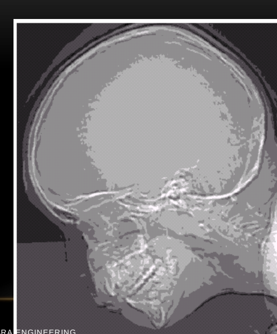
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

INTENSITY LEVEL RESOLUTION (CONT…)



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

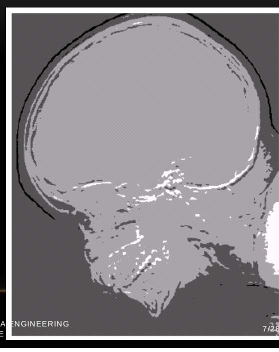
INTENSITY LEVEL RESOLUTION (CONT…)



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

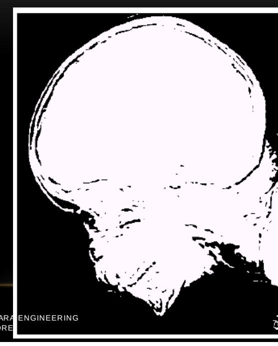
INTENSITY LEVEL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



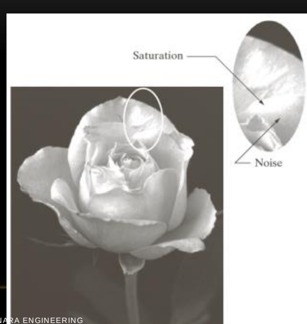
INTENSITY LEVEL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



SATURATION & NOISE

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



RESOLUTION: HOW MUCH IS ENOUGH?

- The big question with resolution is always *how much is enough*?
  - This all depends on what is in the image and what you would like to do with it
  - Key questions include
    - Does the image look aesthetically pleasing?
    - Can you see what you need to see within the image?

RESOLUTION: HOW MUCH IS ENOUGH? (CONT…)

- The picture on the right is fine for counting the number of cars, but not for reading the number plate

INTENSITY LEVEL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Low Detail | Medium Detail | High Detail

INTENSITY LEVEL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

INTENSITY LEVEL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

INTENSITY LEVEL RESOLUTION (CONT…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

SUMMARY

- We have looked at:
  - Human visual system
  - Light and the electromagnetic spectrum
  - Image representation
  - Image sensing and acquisition
  - Sampling, quantisation and resolution
- Next time we start to look at techniques for image enhancement

Aliasing and Moiré Pattern

- All signals (functions) can be shown to be made up of a linear combination sinusoidal signals (sines and cosines) of different frequencies.
- For physical reasons, there is a highest frequency component in all real world signals.
- Theoretically,
  - if a signal is sampled at more than twice its highest frequency component, then it can be reconstructed exactly from its samples.
  - But, if it is sampled at less than that frequency (called undersampling), then aliasing will result.
  - This causes frequencies to appear in the sampled signal that were not in the original signal.
  - The Moiré pattern: The vertical low frequency pattern is a new frequency not in the original patterns.

Aliasing and Moiré Pattern
The effect of aliased frequencies

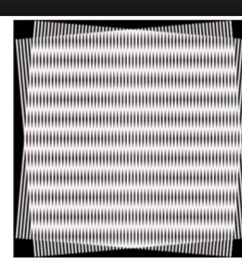


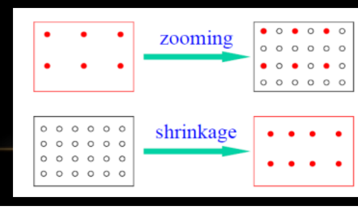
FIGURE 2.24 Illustration of the Moiré pattern effect.

Aliasing and Moiré Pattern

- Note that subsampling of a digital image will cause undersampling if the subsampling rate is less than twice the maximum frequency in the digital image.
- Aliasing can be prevented if a signal is filtered to eliminate high frequencies so that its highest frequency component will be less than twice the sampling rate.
- Gating function: exists for all space (or time) and has value zero everywhere except for a finite range of space/time. Often used for theoretical analysis of signals. But, a gating signal is mathematically defined and contains unbounded frequencies.
- A signal which is periodic, $x(t) = x(t+T)$ for all t and where T is the period, has a finite maximum frequency component. So it is a bandlimited signal.
- Sampling at a higher sampling rate (usually twice or more) than necessary to prevent aliasing is called oversampling.

Zooming and Shrinking Digital Images

- Zooming: increasing the number of pixels in an image so that the image appears larger
  - Nearest neighbor interpolation
    - For example: pixel replication--to repeat rows and columns of an image
  - Bilinear interpolation
    - Smoother
  - Higher order interpolation
- Image shrinking: subsampling



Zooming and Shrinking Digital Images

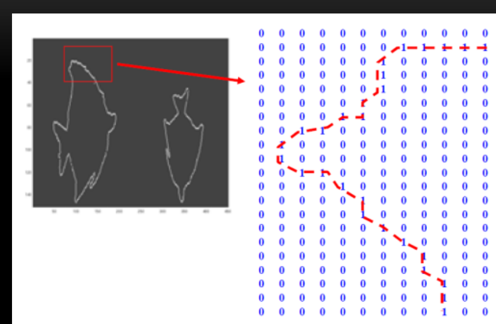
Nearest neighbor Interpolation (Pixel replication)

Bilinear interpolation

a b c
d e f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Relationships Between Pixels



RELATIONSHIP BETWEEN PIXELS

- Neighbours
- Adjacency
- Path
- Connectivity
- Region
- Boundary
- Distance

Some Basic Relationships Between Pixels

- Neighbors of a pixel
  - There are three kinds of neighbors of a pixel:
    - $N_4(p)$ 4-neighbors: the set of horizontal and vertical neighbors
    - $N_D(p)$ diagonal neighbors: the set of 4 diagonal neighbors
    - $N_8(p)$ 8-neighbors: union of 4-neighbors and diagonal neighbors

	O	
O	X	O
	O	

O		O
	X	
O		O

O	O	O
O	X	O
O	O	O

NEIGHBORS OF A PIXEL

- A pixel p at coordinates (x,y) has four *horizontal* and *vertical* neighbors whose coordinates are given by:

  (x+1,y), (x-1, y), (x, y+1), (x,y-1)

	(x, y-1)	
(x-1, y)	*P (x,y)*	(x+1, y)
	(x, y+1)	

This set of pixels, called the 4-*neighbors* or *p*, is denoted by $N_4(p)$. Each pixel is one unit distance from (x,y) and some of the neighbors of p lie outside the digital image if (x,y) is on the border of the image.

NEIGHBORS OF A PIXEL

- The four *diagonal* neighbors of *p* have coordinates:

  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

(x-1, y+1)		(x+1, y-1)
	P (x,y)	
(x-1, y-1)		(x+1, y+1)

and are denoted by $N_D(p)$.

These points, together with the 4-neighbors, are called the 8-neighbors of p, denoted by $N_8(p)$.

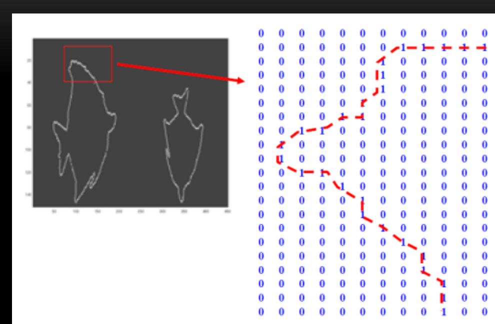
(x-1, y+1)	(x, y-1)	(x+1, y-1)
(x-1, y)	*P (x,y)*	(x+1, y)
(x-1, y-1)	(x, y+1)	(x+1, y+1)

As before, some of the points in $N_D(p)$ and $N_8(p)$ fall outside the image if (x,y) is on the border of the image.

ADJACENCY AND CONNECTIVITY

- Two pixels are adjacent if they are neighbors and their gray levels are similar
- Let *V*: a set of intensity values used to define adjacency and connectivity.
- In a binary image, $V = \{1\}$, if we are referring to adjacency of pixels with value 1.
- In a gray-scale image, the idea is the same, but *V* typically contains more elements, for example, $V = \{180, 181, 182, \ldots, 200\}$
- If the possible intensity values 0 – 255, *V* set can be any subset of these 256 values.

Binary Image Adjacency Between Pixels



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TYPES OF ADJACENCY

1. **4-adjacency:** Two pixels *p* and *q* with values from *V* are 4-adjacent if *q* is in the set $N_4(p)$.
2. **8-adjacency:** Two pixels *p* and *q* with values from *V* are 8-adjacent if *q* is in the set $N_8(p)$.
3. **m-adjacency =(mixed)**

TYPES OF ADJACENCY

- **m-adjacency:**

  Two pixels *p* and *q* with values from *V* are m-adjacent if :
  - *q* is in $N_4(p)$ or
  - *q* is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixel whose values are from V (no intersection)

- **Important Note:** the type of adjacency used must be specified

TYPES OF ADJACENCY

- Mixed adjacency is a modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used.
- For example:

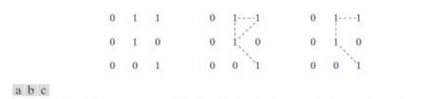
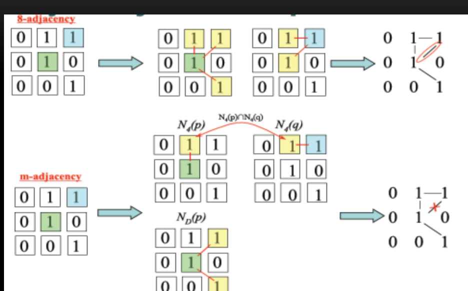


FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

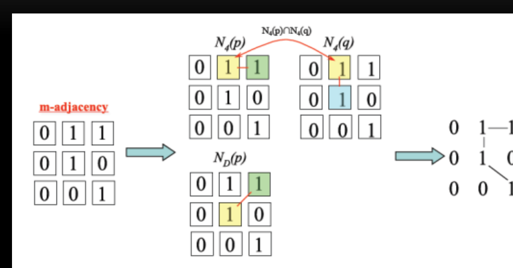
TYPES OF ADJACENCY…

- In this example, we can note that to connect between two pixels (finding a path between two pixels):
  - In 8-adjacency way, you can find multiple paths between two pixels
  - While, in m-adjacency, you can find only one path between two pixels
- So, m-adjacency has eliminated the multiple path connection that has been generated by the 8-adjacency.

ADJACENCY: AN EXAMPLE



M-ADJACENCY EXAMPLE



ADJACENCY, CONNECTIVITY

4-adjacency: Two pixels p and q with the values from set 'V' are 4-adjacent if q is in the set of $N_4(p)$.

e.g. V = { 0, 1}

1	1	0
1	1	0
1	0	1

p in RED color
q can be any value in GREEN color.

ADJACENCY, CONNECTIVITY

8-adjacency: Two pixels p and q with the values from set 'V' are 8-adjacent if q is in the set of $N_8(p)$.

e.g. V = { 1, 2}

0	1	1
0	2	0
0	0	1

p in RED color
q can be any value in GREEN color

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if
(i) q is in $N_4(p)$ OR

(ii) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. V = { 1 }

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if
(i) q is in $N_4(p)$

e.g. V = { 1 }
(i) b & c

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if
(i) q is in $N_4(p)$

e.g. V = { 1 }
(i) b & c

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

Soln: b & c are m-adjacent.

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if
(i) q is in $N_4(p)$

e.g. V = { 1 }
(ii) b & e

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if
(i) q is in $N_4(p)$

e.g. V = { 1 }
(ii) b & e

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

Soln: b & e are m-adjacent.

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if
(i) q is in $N_4(p)$ OR

e.g. V = { 1 }
(iii) e & i

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. V = { 1 }

(iii) e & i

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. V = { 1 }

(iii) e & i

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

Soln: e & i are m-adjacent.

ADJACENCY, CONNECTIVITY

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$ OR

(ii) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. V = { 1 }

(iv) e & c

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

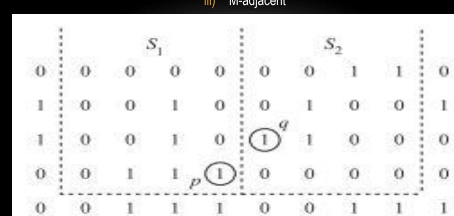
SUBSET ADJACENCY

Two subsets S1 and S2 are adjacent, if some pixel in S1 is adjacent to some pixel in S2. Adjacent means, either 4-, 8- or m-adjacency.

Example:

For V=1, Determine whether these two subsets are

i) 4 Adjacent
ii) 8 Adjacent
iii) M-adjacent



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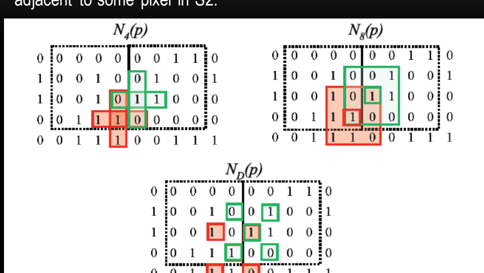
SUBSET ADJACENCY

- Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2.



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A DIGITAL PATH

- A digital path (or curve) from pixel *p* with coordinate (x,y) to pixel *q* with coordinate (s,t) is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1), …, (x_n, y_n) where $(x_0,y_0) = (x,y)$ and $(x_n, y_n) = (s,t)$ and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$
- n is the length of the path
- If $(x_0,y_0) = (x_n, y_n)$, the path is closed.
- We can specify 4-, 8- or m-paths depending on the type of adjacency specified.

A DIGITAL PATH

- Return to the previous example:

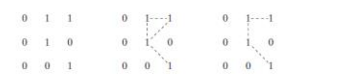


FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

In figure (b) the paths between the top right and bottom right pixels are 8-paths. And the path between the same 2 pixels in figure (c) is m-path

CONNECTIVITY

- Let *S* represent a subset of pixels in an image, two pixels *p* and *q* are said to be connected in *S* if there exists a path between them consisting entirely of pixels in *S*.
- For any pixel *p* in *S*, the set of pixels that are connected to it in *S* is called a *connected component* of *S*. If it only has one connected component, then set *S* is called a *connected set.*

PATHS

Example # 1: Consider the image segment shown in figure. Compute length of the ***shortest-4, shortest-8 & shortest-m paths*** between pixels p & q where,
V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2 →	1	2	3

PATHS

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2 →	1	2	3

PATHS

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2 →	1 ↑	2	3

PATHS

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p ~~2~~	~~1~~	2	3	

PATHS

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p ~~2~~	~~1~~	2	3	

PATHS

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p ~~2~~	~~1~~	2	3	

So, Path does not exist.

PATHS

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p 2	1	2	3	

PATHS

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p ~~2~~	1	2	3	

PATHS

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p ~~2~~	1	2	3	

PATHS

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

So, shortest-8 path = 4

PATHS

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

PATHS

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

So, shortest-m path = 5

PATHS

Example # 2:

Find Shortest 4, 8, -m path:

V = {3, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

REGION AND BOUNDARY

- **Region**
  - Let *R* be a subset of pixels in an image
  - R is called a region if every pixel in R is connected to any other pixel in R.
- **Boundary**

  The *boundary* (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*.

0	1	1	0
0	1	1	1
0	0	1	0
0	0	0	0

0	1	1	0
0	1	1	1
0	0	1	0
1	0	0	0

REGION AND BOUNDARY

If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns in the image.

This extra definition is required because an image has no neighbors beyond its borders

Normally, when we refer to a region, we are referring to subset of an image, and any pixels in the boundary of the region that happen to coincide with the border of the image are included implicitly as part of the region boundary.

REGIONS & BOUNDARIES

Regions that are not adjacent are said to be disjoint.

We consider 4- and 8- adjacency when referring to regions.

Below regions are adjacent only if 8-adjacency is used.

1	1	1	
1	0	1	R_i
0	1	0	
0	0	1	
1	1	1	R_j
1	1	1	

REGIONS & BOUNDARIES

Boundaries (border or contour): The boundary of a region R is the set of points that are adjacent to points in the compliment of R.

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

RED colored 1 is NOT a member of border if 4-connectivity is used between region and background. It is if 8-connectivity is used.

Distance measures

- For pixels p, q and z, with coordinates (x,y), (s,t) and (v,w), respectively, D is a distance function if:

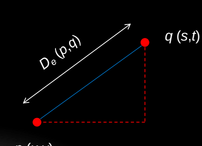
$$(a)\ D(p,q) \geq 0$$
$$(b)\ D(p,q) = D(q,p), \text{ and}$$
$$(c)\ D(p,z) \leq D(p,q) + D(q,z)$$

- The Euclidean distance $D_e(p, q)$ $D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$
- The city-block (Manhattan) distance $D_4(p, q)$ $D_4(p,q) = |x-s| + |y-t|$
- The chessboard distance $D_8(p, q)$ $D_8(p,q) = \max(|x-s|, |y-t|)$

DISTANCE MEASURES

- The *Euclidean Distance* between p and q is defined as:
  $D_e(p,q) = [(x - s)^2 + (y - t)^2]^{1/2}$

Pixels having a distance less than or equal to some value r from (x,y) are the points contained in a disk of radius r centered at (x,y)



CITY BLOCK DISTANCE OR D4 DISTANCE

- It's called city-block distance, because it is calculated as if on each pixel between your two coordinates stood a block (house) which you have to go around.
- That means, you can only go along the vertical or horizontal lines between the pixels but not diagonal. It's the same like the movement of the rook on a chess field.

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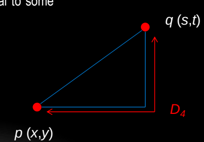
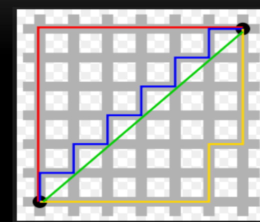
CITY BLOCK DISTANCE OR D4 DISTANCE

- The D_4 *distance* (also called *city-block distance*) between *p* and *q* is defined as:
  $D_4\,(p,q) = |\,x - s\,| + |\,y - t\,|$

Pixels having a D_4 distance from (x,y), less than or equal to some value r form a Diamond centered at (x,y)



CITY BLOCK DISTANCE AND EUCLIDEAN DISTANCE



- Green line is **Euclidean distance.** Red, blue and yellow are **City Block Distance**

DISTANCE MEASURES- D_4

Example:

The pixels with distance $D_4 \le 2$ from (x,y) form the following contours of constant distance.

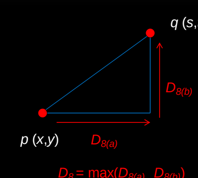
The pixels with D_4 = 1 are the 4-neighbors of (x,y)

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

CHESSBOARD DISTANCE OR D_8 DISTANCE

- The D_8 *distance* (also called *chessboard distance*) between *p* and *q* is defined as:
  $D_8\,(p,q) = \max(|\,x - s\,|, |\,y - t\,|)$

Pixels having a D_8 distance from (x,y), less than or equal to some value r form a square Centered at (x,y)



DISTANCE MEASURES- D_8

Example:

D_8 distance $\le$ 2 from (x,y) form the following contours of constant distance.

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

DM DISTANCE

- **Dm distance:**

  is defined as the shortest m-path between the points.

  In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.

DISTANCE MEASURES

- Example:
- Consider the following arrangement of pixels and assume that p, p2, and p4 have value 1 and that p1 and p3 can have can have a value of 0 or 1
- Suppose that we consider the adjacency of pixels values 1 (i.e. V = {1})

	p_3	p_4
p_1	p_2	
p		

DISTANCE MEASURES

- Cont. Example:

Now, to compute the D_m between points p and p_4

Here we have 4 cases:

Case1: If p_1 =0 and p_3 = 0

The length of the shortest m-path

(the D_m *distance)* is 2 *(p, p_2, p_4)*

	p_3	p_4
p_1	p_2	
p		

	0	1
0	1	
1		

DISTANCE MEASURES

- Cont. Example:

Case2: If p_1 =1 and p_3 = 0

now, *p_2 and p will no longer be adjacent (see m-adjacency definition)*

then, the length of the shortest

path will be 3 (p, p_1, p_2, p_4)

	p_3	p_4
p_1	p_2	
p		

	0	1
1	1	
1		

DISTANCE MEASURES

- Cont. Example:

Case3: If p_1 =0 and p_3 = 1

The same applies here, and the shortest –m-path will be 3 (p, p_2, p_3, p_4)

	p_3	p_4
p_1	p_2	
p		

	1	1
0	1	
1		

DISTANCE MEASURES

- Cont. Example:

Case4: If p_1 =1 and p_3 = 1

The length of the shortest m-path will be 4 (p, p_1 , p_2, p_3, p_4)

	p_3	p_4
p_1	p_2	
p		

	1	1
1	1	
1		

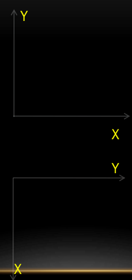
LINEAR & NON-LINEAR OPERATIONS

$$H(af+bh)=aH(f)+bH(g)$$

- Where H is an operator whose input and output are images.
- f and g are two images a and b are constants.

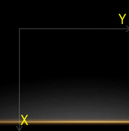
- H is said to be linear operation if it satisfies the above equation or else H is a nonlinear operator.

NEIGHBORS OF A PIXEL



NEIGHBORS OF A PIXEL

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) & f(0,4) & ----- \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) & f(1,4) & ----- \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) & f(2,4) & ----- \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) & f(3,4) & ----- \\ | & | & | & | & | & ----- \\ | & | & | & | & | & ----- \end{bmatrix}$$



NEIGHBORS OF A PIXEL

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) & f(0,4) & ----- \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) & f(1,4) & ----- \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) & f(2,4) & ----- \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) & f(3,4) & ----- \\ | & | & | & | & | & ----- \\ | & | & | & | & | & ----- \end{bmatrix}$$

- A Pixel p at coordinates (x, y) has 4 horizontal and vertical neighbors.
- Their coordinates are given by:

  (x+1, y) (x-1, y) (x, y+1) & (x, y-1)

  f(2,1) f(0,1) f(1,2) f(1,0)
- This set of pixels is called the ***<u>4-neighbors</u>*** of p denoted by $N_4(p)$.
- Each pixel is unit distance from (x ,y).

NEIGHBORS OF A PIXEL

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) & f(0,4) & ----- \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) & f(1,4) & ----- \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) & f(2,4) & ----- \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) & f(3,4) & ----- \\ | & | & | & | & | & ----- \\ | & | & | & | & | & ----- \end{bmatrix}$$

- A Pixel p at coordinates (x, y) has 4 diagonal neighbors.
- Their coordinates are given by:

  (x+1, y+1) (x+1, y-1) (x-1, y+1) & (x-1, y-1)

  f(2,2) f(2,0) f(0,2) f(0,0)
- This set of pixels is called the ***<u>diagonal-neighbors</u>*** of p denoted by $N_D(p)$.
- diagonal neighbors + 4-neighbors = 8-neighbors of p.
- They are denoted by $N_8(p)$. So, $N_8(p) = N_4(p) + N_D(p)$

ADJACENCY, CONNECTIVITY

<u>Adjacency:</u> Two pixels are adjacent if they are neighbors and their intensity level 'V' satisfy some specific criteria of similarity.

e.g. V = {1}
V = { 0, 2}
Binary image = { 0, 1}
Gray scale image = { 0, 1, 2, ------, 255}

In binary images, 2 pixels are adjacent if they are neighbors & have some intensity values either 0 or 1.

In gray scale, image contains more gray level values in range 0 to 255.

ADJACENCY, CONNECTIVITY

<u>m-adjacency:</u> Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$ OR

(ii) q is in $N_D(p)$ & the set <u>$N_4(p)$ ∩ $N_4(q)$</u> have no pixels whose values are from 'V'.

e.g. V = { 1 }

(iv) e & c

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

<u>Soln:</u> e & c are NOT m-adjacent.

ADJACENCY, CONNECTIVITY

Connectivity: 2 pixels are said to be connected if their exists a path between them.

Let 'S' represent subset of pixels in an image.

Two pixels p & q are said to be connected in 'S' if their exists a path between them consisting entirely of pixels in 'S'.

For any pixel p in S, the set of pixels that are connected to it in S is called a **connected component of S**.

PATHS

Paths: A path from pixel p with coordinate (x, y) with pixel q with coordinate (s, t) is a sequence of distinct sequence with coordinates $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ where

$(x, y) = (x_0, y_0)$
& $(s, t) = (x_n, y_n)$

Closed path: $(x_0, y_0) = (x_n, y_n)$

DISTANCE MEASURES

Distance Measures: Distance between pixels p, q & z with co-ordinates (x, y), (s, t) & (v, w) resp. is given by:

a) $D(p, q) \geq 0$ [$D(p, q) = 0$ if $p = q$] ....called reflexivity
b) $D(p, q) = D(q, p)$ ....called symmetry
c) $D(p, z) \leq D(p, q) + D(q, z)$ ....called transmitivity

Euclidean distance between p & q is defined as-

$$D_e(p, q) = [(x - s)^2 + (y - t)^2]^{1/2}$$

DISTANCE MEASURES

City Block Distance: The D_4 distance between p & q is defined as

$$D_4(p, q) = |x - s| + |y - t|$$

In this case, pixels having D_4 distance from (x, y) less than or equal to some value r form a diamond centered at (x, y).

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Pixels with D_4 distance $\leq$ 2 forms the following contour of constant distance.

DISTANCE MEASURES

Chess-Board Distance: The D_8 distance between p & q is defined as

$$D_8(p, q) = \max(|x - s|, |y - t|)$$

In this case, pixels having D_8 distance from (x, y) less than or equal to some value r form a square centered at (x, y).

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with D_8 distance $\leq$ 2 forms the following contour of constant distance.

End of Unit 2