Multicarrier Modulation, OFDMA and SC-FDMA
&
Multiple Antenna Transmission and Reception

MODULE – 2

SYLLABUS- MODULE 2

Part 1

- **Multicarrier Modulation:** OFDM basics, OFDM in LTE, Timing and Frequency Synchronization, PAR, SC-FDE (Sec 3.2 – 3.6 of Text).
- **OFDMA and SC-FDMA**:OFDM with FDMA,TDMA,CDMA, OFDMA, SC-FDMA, OFDMA and SC-FDMA in LTE (Sec 4.1 – 4.3, 4.5 of Text).

Part 2

- **Multiple Antenna Transmission and Reception:** Spatial Diversity overview, Receive Diversity, Transmit Diversity, Interference cancellation and signal enhancement, Spatial Multiplexing, Choice between Diversity, Interference suppression and Spatial Multiplexing (Sec 5.1 – 5.6 of Text).

PART-1

INTRODUCTION TO MULTICARRIER MODULATION TECHNIQUES

The unifying common feature of multicarrier modulation techniques is the use of ***multiple parallel subcarriers***, invariably generated by the (inverse) discrete Fourier transform.

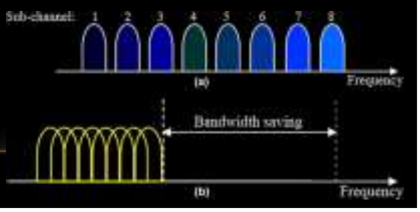
- Most common type of multicarrier modulation is Orthogonal Frequency Division Multiplexing (***OFDM***) and Single-Carrier Frequency Division Multiple Access (***SC-FDMA***).
- Used in:
  - ✓ *Digital Subscriber Lines (DSL),*
  - ✓ *Wireless LANs (802.11a/g/n),*
  - ✓ *Digital Video Broadcasting, and*
  - ✓ *most recently, **beyond 3G cellular technologies such as WiMAX and LTE***

THE MULTICARRIER CONCEPT

- ***The basic idea*** : ***high data rates and intersymbol interference (ISI) free channels***.
- In order to have a channel that does not have ISI,
  - ➢ the symbol time T_s has to be larger
    - ➢—often significantly larger than the channel delay spread τ.
- Digital communication ***systems*** simply ***cannot function if ISI is present***
  - ➢ an error floor quickly develops and as T_s approaches or falls below τ, ***the bit error rate (BER) becomes intolerable***

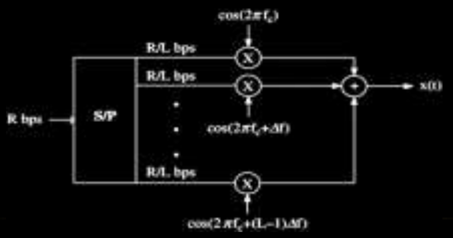
THE MULTICARRIER CONCEPT…

- In order ***to overcome high BER***, multicarrier modulation ***divides*** the ***high-rate transmit bitstream into L lower-rate substreams***,
  - ➢ where L is chosen so that *each* of the subcarriers has effective symbol time $T_sL >> \tau$, and is hence effectively ISI-free.
- These individual ***sub-streams can then be sent over L parallel subcarriers***
  - ➢ maintaining the total desired data rate



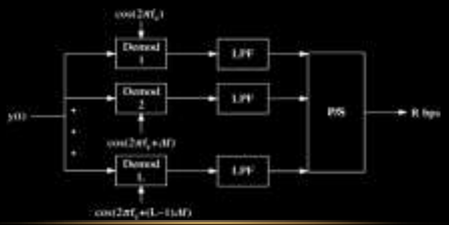
AN ELEGANT APPROACH TO INTERSYMBOL INTERFERENCE

A basic multicarrier transmitter: a high-rate stream of *R* bps is broken into *L* parallel streams each with rate *R*/*L* and then multiplied by a different carrier frequency.



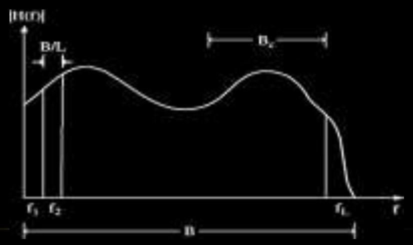
AN ELEGANT APPROACH TO INTERSYMBOL INTERFERENCE…..

- A basic multicarrier receiver: each subcarrier is decoded separately, requiring *L* independent receivers.



AN ELEGANT APPROACH TO ISI…..

The transmitted multicarrier signal experiences approximately ***flat fading*** on each subcarrier since B/L << Bc, even though the overall channel experiences frequency selective fading, that is, B > Bc.



LIMITATIONS

- In a realistic implementation, a ***large bandwidth penalty will be inflicted*** since
  - The ***subcarriers can't have perfectly rectangular pulse shapes*** and still be time-limited.
  - ***very high-quality*** (***and hence, expensive***), ***low-pass filters will be required*** to ***maintain the orthogonally*** of the subcarriers at the receiver.
  - Most importantly, this scheme ***requires L independent RF units*** and ***demodulation paths***.

OFDM BASICS

- OFDM employs an efficient computational technique known as the Fast Fourier Transform (FFT).

$$\mathrm{DFT}\{x[n]\} = X[m] \triangleq \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} x[n] e^{-j\frac{2\pi nm}{L}},$$

$$\mathrm{IDFT}\{X[m]\} = x[n] \triangleq \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} X[m] e^{j\frac{2\pi nm}{L}},$$

- Each ***input symbol X[m]*** is simply ***scaled by a complex-value H[m]***
- So, given knowledge of the channel frequency response *H*[*m*] at the receiver, it is trivial to recover the input symbol by simply computing

$$\hat{X}[m] = \frac{Y[m]}{H[m]},$$

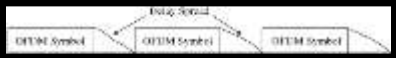
- In principle, the ***ISI***—which is the most serious form of interference in a wideband channel—***has been mitigated***.

BLOCK TRANSMISSION WITH GUARD INTERVALS

- In order to ***keep each OFDM symbol independent of the others*** after going through a ***wireless channel***, it is necessary to ***introduce a guard time*** in between each OFDM symbol, as shown here:

OFDM Symbol	guard	OFDM Symbol	guard	OFDM Symbol

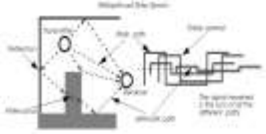
- This way, after receiving a series of OFDM symbols, ***as long as the guard time T_g is larger than the delay spread of the channel τ***, ***each OFDM symbol will only interfere with itself***.

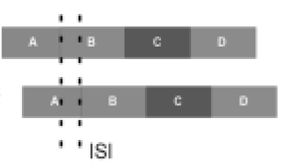


- Put simply, OFDM transmissions allow ISI *within* an OFDM symbol, but by including a sufficiently large guard band, it is possible to guarantee that there is no interference *between* subsequent OFDM symbols.

THE CYCLIC PREFIX

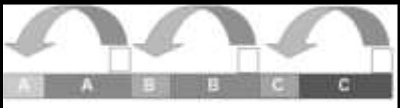
- The ***cyclic prefix acts as a buffer region*** or guard interval to protect the OFDM signals from ***inter symbol interference***.
- We have learned that we minimize the ISI a lot when we make the size of the symbol larger (the size of the delay spread becomes relatively minor compared to the larger size).
- But as much as we could increase this symbol size, the effects of ISI would always be present.





THE CYCLIC PREFIX....

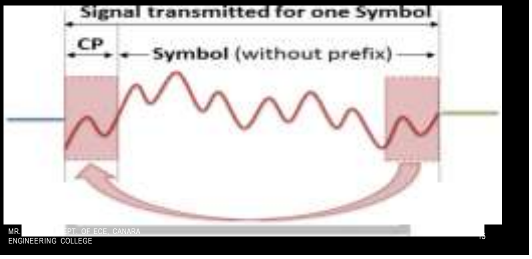
- To permanently eliminate this problem, the solution is to find a way where the 'lost' part of the symbol could be 'recovered'.
- The ***one way to do this*** is ***by copying or duplicating an initial part of the symbol, and inserting the end of it*** and ***it acts as Cyclic Prefix***.



- *CP: **copy*** a small part of the initial information (hence the name prefix) to the end of each symbol (hence the name cyclic).
- Thus, the ***receiver can identify the end points*** of each symbol and ***correctly correlate the information***, thereby eliminating the interference problem.

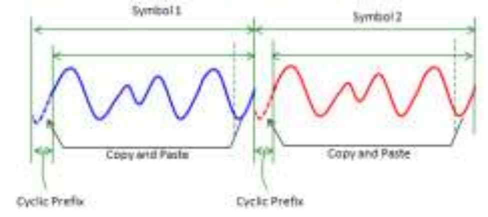
THE CYCLIC PREFIX....

- The ***receiver already "knows" the last part of the symbol*** at the time it receives the first component of the signal, the multipath shortest path.
- In this case, it can make the correlation with the information of other multipath components, ***making the corresponding correlations and getting complete information***.



THE CYCLIC PREFIX....

- In addition, the ***CP also helps to make an initial estimation*** of ***time and frequency synchronism***, using the same reasoning correlation of known information that arrives over time.

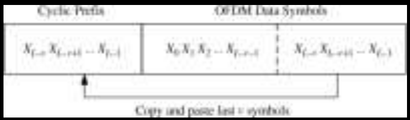


TYPES OF CYCLIC PREFIX

- In LTE, two types of CPs are defined.
- Normal and Extended.
- The ***normal cyclic prefix*** is intended to be sufficient for the majority of scenarios
- the ***extended cyclic prefix*** is intended for scenarios with **particularly high delay spread**.

THE OFDM CYCLIC PREFIX.

- In order for the IFFT/FFT to create an ISI-free channel, the channel must appear to provide a circular convolution.
- If a cyclic prefix is added to the transmitted signal, as shown in figure, then this creates a signal that appears to be $x[n]_L$, and so $y[n] = x[n]$ $h[n]$.



THE OFDM CYCLIC PREFIX.

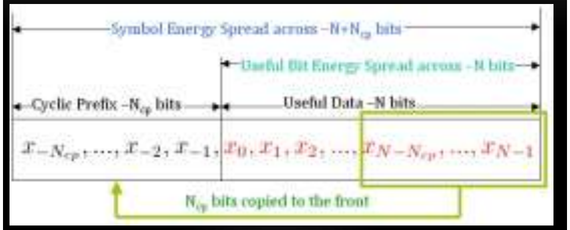
- Let's see how this works.
- If the ***maximum channel delay spread*** has a duration of v + 1 samples, then by ***adding a guard band*** of at least v samples between OFDM symbols, each OFDM symbol is made independent of those coming before and after it,
- and so just a single OFDM symbol can be considered.
- Representing such an OFDM symbol in the time domain as a length L vector gives

$$\mathbf{x} = [x_1\ x_2 \ldots x_L].$$

- After applying a cyclic prefix of length v, the actual transmitted signal is

$$\mathbf{x}_{cp} = [\underbrace{x_{L-v}\ x_{L-v+1}\ \cdots\ x_{L-1}}_{\text{Cyclic prefix}}\ \underbrace{x_0\ x_1\ \cdots\ x_{L-1}}_{\text{OFDM data}}]$$

THE OFDM CYCLIC PREFIX...



THE OFDM CYCLIC PREFIX...

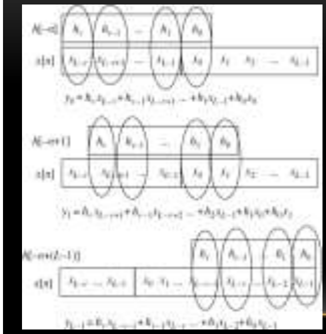
- The output of the channel is by definition $\mathbf{y}_{cp} = \mathbf{h} * \mathbf{x}_{cp}$,
  - where **h** is a length $v + 1$ vector describing the impulse response of the channel during the OFDM symbol.
- The output $\mathbf{y}_{cp}$ has $(L + v) + (v + 1) - 1 = L + 2v$ samples.
- The first v samples of $\mathbf{y}_{cp}$ contain interference from the preceding OFDM symbol, and so are discarded.
- The last v samples disperse into the subsequent OFDM symbol, and so also are discarded.
- This leaves exactly L samples for the desired output **y**, which is precisely what is required to recover the L data symbols embedded in **x**.
- These L samples of **y** will be equivalent to $\mathbf{y} = \mathbf{h} \circledast \mathbf{x}$.

THE OFDM CYCLIC PREFIX...

- Consider for the moment just y_0, that is, the first element in **y**.
- Due to the cyclic prefix, y_0 depends on x_0 and the circularly wrapped values $x_{L-v} \ldots x_{L-1}$.
- That is:

$$\begin{aligned} y_0 &= h_0x_0 + h_1x_{L-1} + \cdots + h_vx_{L-v} \\ y_1 &= h_0x_1 + h_1x_0 + \cdots + h_vx_{L-v+1} \\ &\vdots \\ y_{L-1} &= h_0x_{L-1} + h_1x_{L-2} + \cdots + h_vx_{L-v-1} \end{aligned}$$

THE OFDM CYCLIC PREFIX...



- The OFDM cyclic prefix creates a ***circular convolution*** at the receiver (signal y) even though the actual channel causes a linear convolution

THE OFDM CYCLIC PREFIX...

- It can be seen that this is exactly the value of $y_0, y_1, \ldots, y_{L-1}$ resulting from $\mathbf{y} = \mathbf{x} \circledast \mathbf{h}$.
- Thus, by mimicking a circular convolution, a cyclic prefix that is at least as long as the channel duration allows
  - the channel output **y** to be decomposed into a simple multiplication of the channel frequency response **H** = DFT{**h**} and
  - the channel frequency domain input, **X** = DFT{**x**}.

CYCLIC PREFIX DRAWBACKS…

- The cyclic prefix, although elegant and simple, is not entirely free.
- It comes with both a **bandwidth and power penalty**.
- Since *v* redundant symbols are sent, the required ***bandwidth for OFDM increases*** from B to $\frac{L+v}{L}B$
- Similarly, an additional *v* symbols must be counted against the transmit power budget.
- Hence, the ***cyclic prefix carries a power penalty*** $10\log_{10}\frac{L+v}{L}$ dB in addition to the bandwidth penalty.
- In summary, the use of cyclic prefix entails data rate and power losses that are both

$$\text{Rate Loss} = \text{Power Loss} = \frac{L}{L+v}$$

- The ***"wasted" power*** has increased importance in an interference-limited wireless system, since ***it causes interference to neighbouring users***

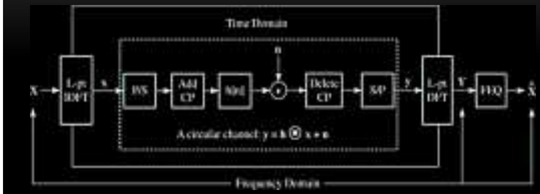
FREQUENCY EQUALIZATION

- In order for the received symbols to be estimated, the complex channel gains for each subcarrier must be known, which corresponds to knowing the ***amplitude and phase of the subcarrier***.
- For simple modulation techniques like ***QPSK that do not use the amplitude to transmit information***, just the ***phase information is sufficient***.
- After the FFT is performed, the ***data symbols*** are **estimated** using a ***one-tap frequency domain equalizer***, or FEQ, as

$$\hat{X}_l = \frac{Y_l}{H_l}$$

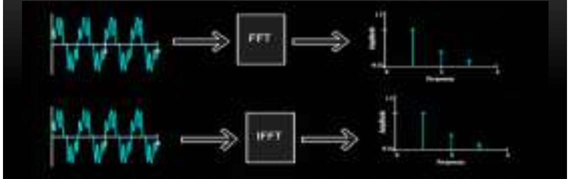
- where H_l is the ***complex*** response of the channel at the frequency $f_c + (l - 1)\Delta f$, and therefore it ***both corrects the phase*** and ***equalizes the amplitude*** before the decision device.

OFDM BLOCK DIAGRAM



- In OFDM, the ***encoding and decoding is done in the frequency domain***
  - where X, Y, and $\hat{X}$ contain the L transmitted, received, and estimated data symbols.

FFT AND IFFT ARE A MATCHED LINEAR PAIR



(a) a time domain signal comes out as a spectrum out of a FFT and IFFT . They both do the same thing.

(b) A frequency domain signal comes out as a time domain signal out of a IFFT.

FFT AND IFFT ARE A MATCHED LINEAR PAIR

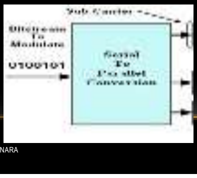
(c) The pair return back the original input.

(d) The pair return back the input no matter what it is.

(e) the pair is commutable so they can be reversed and they will still return the original input.

TRANSMITTER OPERATION:

1. The first step in OFDM is to ***break a wideband signal of Bandwidth B into L narrowband signals*** (**subcarriers**) each of bandwidth *B/L*.
   - This way, the aggregate symbol rate is maintained, but each subcarrier experiences flat fading, or ISI-free communication, as long as a cyclic prefix that exceeds the delay spread is used.
   - The *L* subcarriers for a given OFDM symbol are represented by a vector **X**, which contains the *L* current symbols.



TRANSMITTER OPERATION:

2. In order ***to use a single wideband radio*** instead of *L* independent narrow band radios, the ***subcarriers are created digitally*** using an IFFT operation.

3. In order for the IFFT/FFT to decompose the ISI channel into orthogonal subcarriers, a cyclic prefix of length *v* must be appended after the IFFT operation.

  - The resulting *L* + *v* symbols are then sent in serial through the wideband channel.

RECEIVER OPERATION:

- ***4. At the receiver,*** the ***cyclic prefix is discarded***, and the *L* received symbols are demodulated using an FFT operation, which results in *L* data symbols, each of the form $Y_l = H_l X_l + N_l$ for subcarrier *l*.
- 5. Each subcarrier can then be equalized via an FEQ by simply dividing by the complex channel gain *H*[*i*] for that subcarrier.
  - This results in $\hat{X}_l = X_l + N_l/H_l$

BASEBAND AND PASSBAND SIGNAL

Baseband Signal :

- The ***information signal*** is called the baseband signal.
- The bandwidth is always a ***positive quantity*** so the bandwidth of this signal is ***fm***.

Passband Signal :

- The ***multiplication of the signal with a sinusoid carrier signal*** translates the whole thing up to ***fc*** .
- This signal is now called the passband signal.
- This signal ***extends in range from (-fc -fm) to (fc + fm.).***
- The new signal has ***doubled in bandwidth***.
- The ***passband signal bandwidth is double that of the baseband signal***.

BASEBAND AND PASSBAND SIGNAL

- Baseband becomes Passband by translation to higher frequency .
- The positive frequency spectrum becomes the upper sideband and the negative frequency spectrum become the lower side band

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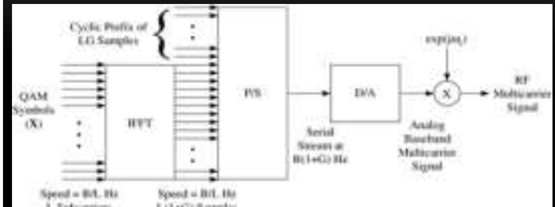
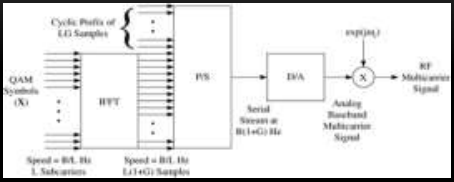
OFDM IN LTE

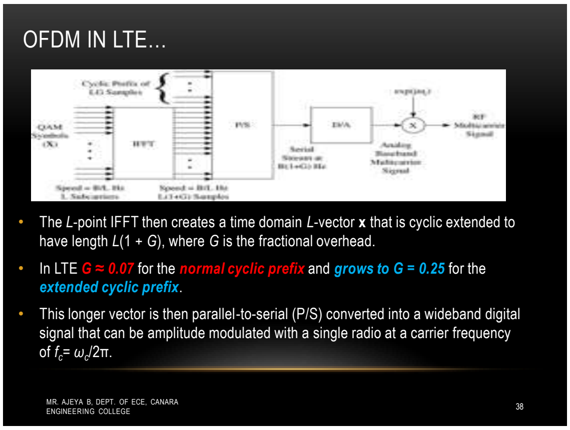


OFDM ***baseband to passband*** transmitter

OFDM IN LTE…

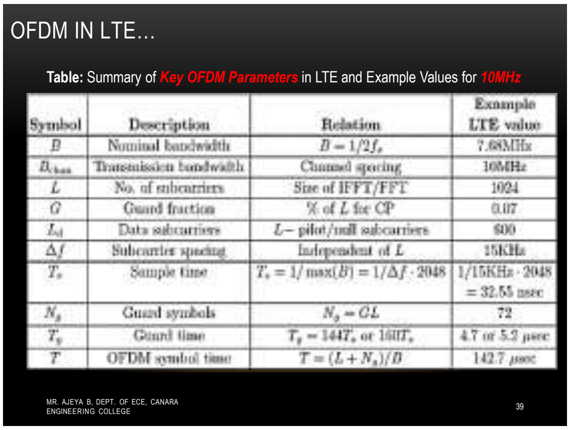


- The ***inputs*** to this figure are *L* independent ***QAM symbols*** (the vector **X**), and these *L* symbols are ***treated as separate subcarriers***.
- These ***L data-bearing symbols*** can be ***created from a bit stream by a symbol mapper*** and ***serial-to-parallel convertor*** (S/P).



OFDM IN LTE…

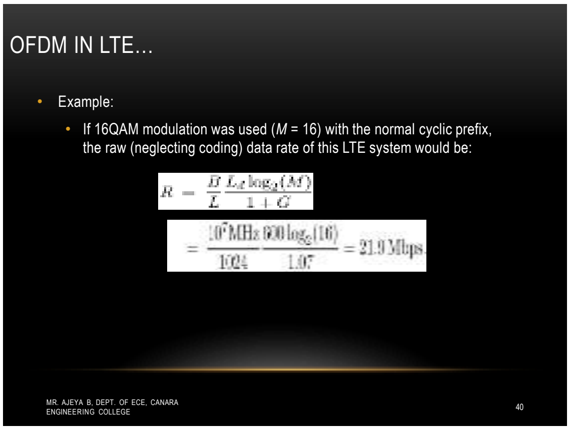
- The *L*-point IFFT then creates a time domain *L*-vector **x** that is cyclic extended to have length *L*(1 + *G*), where *G* is the fractional overhead.
- In LTE ***G ≈ 0.07*** for the ***normal cyclic prefix*** and ***grows to G = 0.25*** for the ***extended cyclic prefix***.
- This longer vector is then parallel-to-serial (P/S) converted into a wideband digital signal that can be amplitude modulated with a single radio at a carrier frequency of $f_c = \omega_c/2\pi$.



OFDM IN LTE…

Table: Summary of ***Key OFDM Parameters*** in LTE and Example Values for ***10MHz***

Symbol	Description	Relation	Example LTE value
B	Nominal bandwidth	$B = 1/2f_s$	7.68MHz
B_{chan}	Transmission bandwidth	Channel spacing	10MHz
L	No. of subcarriers	Size of IFFT/FFT	1024
G	Guard fraction	% of L for CP	0.07
L_d	Data subcarriers	$L-$ pilot/null subcarriers	600
Δf	Subcarrier spacing	Independent of L	15KHz
T_s	Sample time	$T_s = 1/\max(B) = 1/\Delta f \cdot 2048$	1/15KHz · 2048 = 32.55 nsec
N_g	Guard symbols	$N_g = GL$	72
T_g	Guard time	$T_g = 144T_s$ or $160T_s$	4.7 or 5.2 μsec
T	OFDM symbol time	$T = (L + N_g)/B$	142.7 μsec

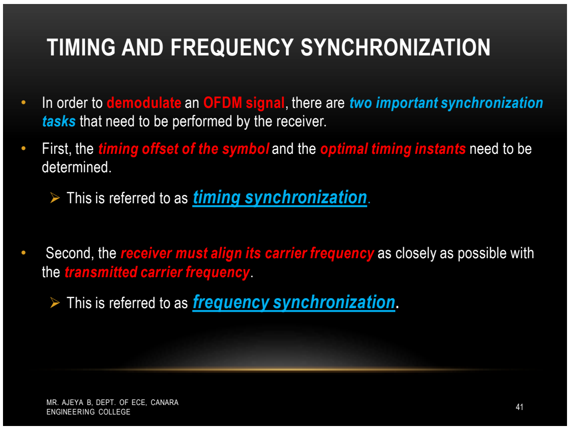


OFDM IN LTE…

- Example:
  - If 16QAM modulation was used (*M* = 16) with the normal cyclic prefix, the raw (neglecting coding) data rate of this LTE system would be:

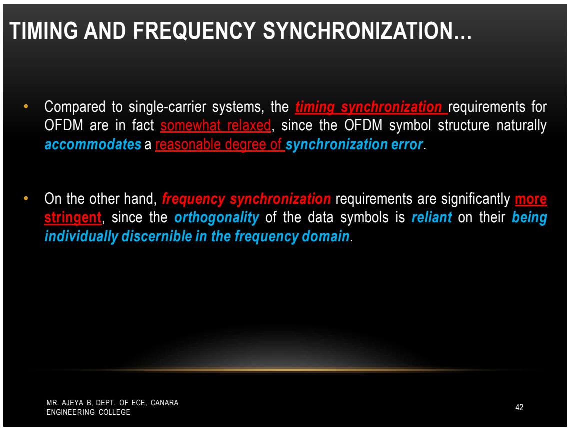
$$R = \frac{B}{L}\frac{L_d \log_2(M)}{1+G}$$

$$= \frac{10^7\text{MHz}}{1024}\frac{600\log_2(16)}{1.07} = 21.9\ \text{Mbps}$$



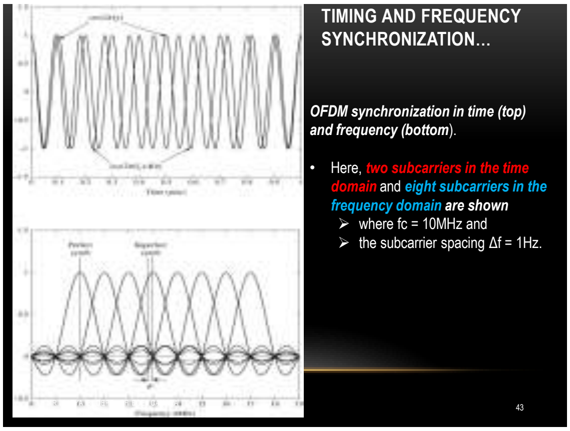
TIMING AND FREQUENCY SYNCHRONIZATION

- In order to **demodulate** an **OFDM signal**, there are ***two important synchronization tasks*** that need to be performed by the receiver.
- First, the ***timing offset of the symbol*** and the ***optimal timing instants*** need to be determined.
  - ➢ This is referred to as ***timing synchronization***.
- Second, the ***receiver must align its carrier frequency*** as closely as possible with the ***transmitted carrier frequency***.
  - ➢ This is referred to as ***frequency synchronization***.



TIMING AND FREQUENCY SYNCHRONIZATION…

- Compared to single-carrier systems, the ***timing synchronization*** requirements for OFDM are in fact somewhat relaxed, since the OFDM symbol structure naturally ***accommodates*** a reasonable degree of ***synchronization error***.
- On the other hand, ***frequency synchronization*** requirements are significantly more stringent, since the ***orthogonality*** of the data symbols is ***reliant*** on their ***being individually discernible in the frequency domain***.



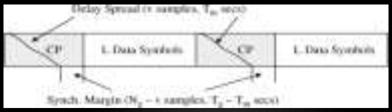
TIMING AND FREQUENCY SYNCHRONIZATION…

OFDM synchronization in time (top) and frequency (bottom).

- Here, ***two subcarriers in the time domain*** and ***eight subcarriers in the frequency domain are shown***
  - ➢ where fc = 10MHz and
  - ➢ the subcarrier spacing Δf = 1Hz.

TIMING SYNCHRONIZATION

- The effect of timing errors in symbol synchronization is somewhat relaxed in OFDM due to the presence of a cyclic prefix.
- In the case that perfect synchronization is not maintained, it is still possible to tolerate a timing offset of *τ* seconds without any degradation in performance as long as $0 \leq \tau \leq T_g - T_m$, where as usual T_g is the guard time (cyclic prefix duration) and T_m is the maximum channel delay spread.
- This acceptable range of *τ* is referred to as the **timing synchronization margin**



If the timing offset τ is not within this window 0 ≤ τ ≤ Tm − Tg, inter-symbol interference (ISI) occurs regardless of whether the phase shift is appropriately accounted for

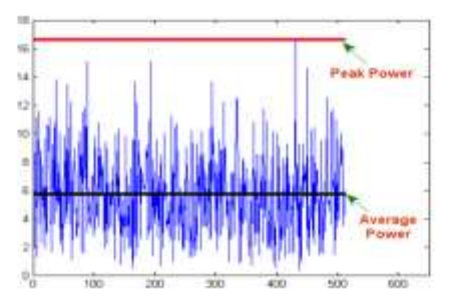
FREQUENCY SYNCHRONIZATION

- OFDM achieves a ***high degree of bandwidth efficiency*** compared to other wideband systems.
- The subcarrier packing is extremely tight compared to conventional modulation techniques…
  - which requires a guard band on the order of 50% or more
  - in addition to special transmitter architectures that suppress the redundant negative-frequency portion of the passband signal.
- ***The price to be paid*** for this bandwidth efficiency is that
  - the ***multicarrier signal is very sensitive to frequency offsets*** due to the fact that the *subcarriers overlap*, rather than having each subcarrier truly spectrally isolated.

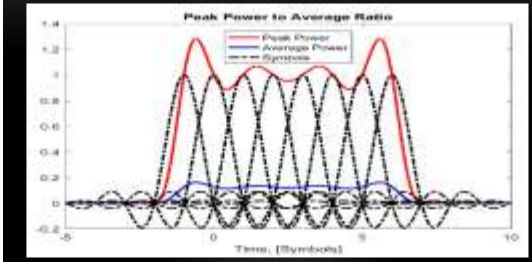
FREQUENCY SYNCHRONIZATION…

- Since the ***zero crossings*** of the frequency domain sinc pulses all line up as long as the frequency offset *δ* = 0, there is no interference between the subcarriers.
- In practice, of course, the frequency offset is not always zero.
- The major causes for this are mismatched oscillators at the transmitter and receiver and Doppler frequency shifts due to mobility.
- Since precise crystal oscillators are expensive, tolerating some degree of frequency offset is essential in a consumer OFDM system like LTE.

THE PEAK-TO-AVERAGE RATIO- PAPR



THE PEAK-TO-AVERAGE RATIO- PAPR



THE PEAK-TO-AVERAGE RATIO- PAPR

- The PAPR is the relation between the ***maximum power*** of a sample in a given OFDM transmit symbol ***divided*** by the ***average power*** of that OFDM symbol.
- In simple terms, ***PAPR is the ratio of peak power to the average power of a signal***.
- This ratio of the peak to average power value is termed as Peak-to-Average Power Ratio, expressed in dB.

THE PEAK-TO-AVERAGE RATIO

➢ **OFDM signals have higher PAPR**

- *The reason* for this is that in the time domain, a multicarrier signal is the sum of many narrowband signals.
  - ➢ At some times, this sum is large;
  - ➢ at other times it is small
    - ➢ which means that the *peak value of the signal is substantially larger than the average value*.

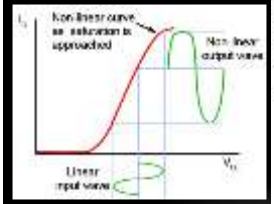
PAR….

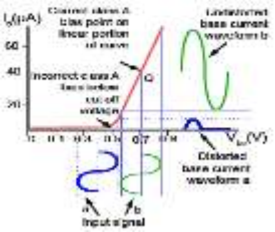
- *High PAR* is one of the most important *implementation challenges* that face OFDM because
  - ➢ it *reduces the efficiency* and hence
  - ➢ *increases* the *cost of the RF power amplifier*, which is one of the most expensive components in the radio.

PAR….

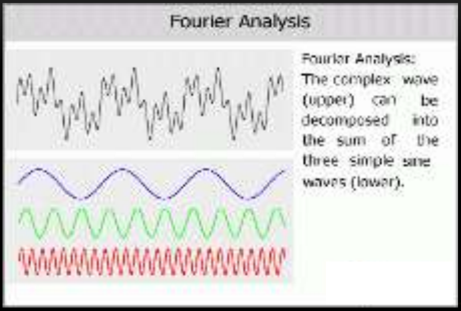
- Alternatively, the same power amplifier can be used but the input power to the PA must be reduced:
  - ➢ this is known as *input backoff (IBO)*
  - ➢ results in a lower average SNR at the receiver, and
  - ➢ hence a *reduced transmit range*

PAR….





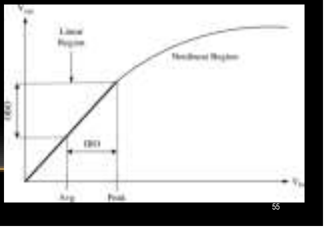
PAR….

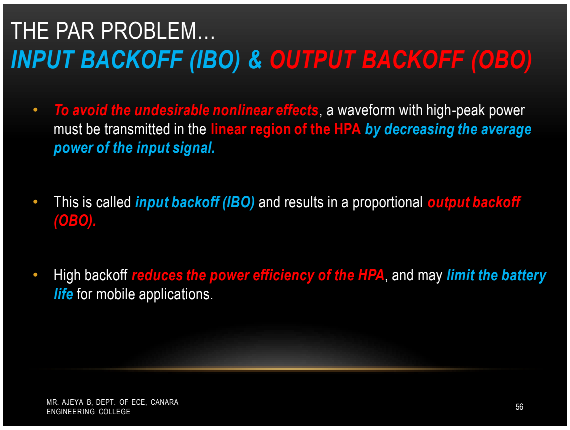

Fourier Analysis

Fourier Analysis: The complex wave (upper) can be decomposed into the sum of the three simple sine waves (lower).

THE PAR PROBLEM

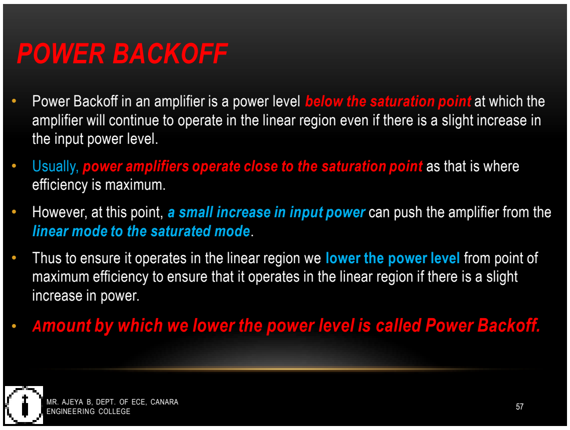
- When a high-peak signal is transmitted through a nonlinear device such as a *high-power amplifier (HPA)* or digital-to-analog converter (DAC)
  - ➢ it generates out-of-band energy (***spectral regrowth***) and in-band distortion (constellation tilting and scattering).
- These degradations may affect the system performance severely.
- The nonlinear behaviour of ***HPA*** can be characterized by amplitude modulation/amplitude modulation (AM/AM) and amplitude modulation/phase modulation (AM/PM) responses.





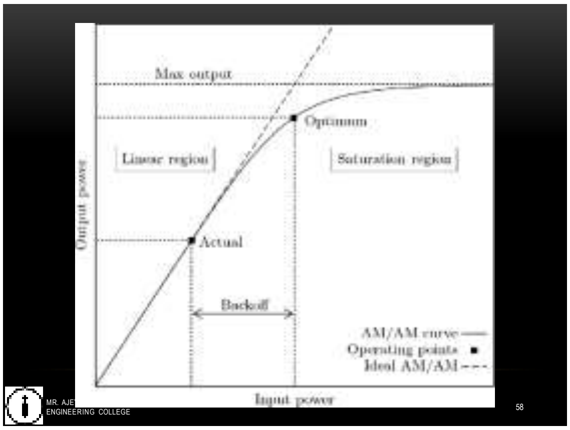
THE PAR PROBLEM…
INPUT BACKOFF (IBO) & OUTPUT BACKOFF (OBO)

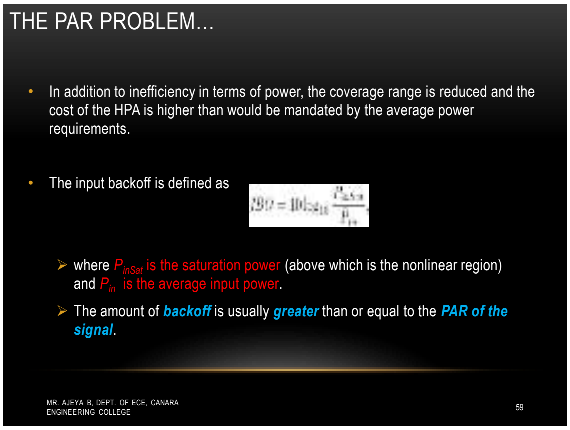
- *To avoid the undesirable nonlinear effects*, a waveform with high-peak power must be transmitted in the linear region of the HPA *by decreasing the average power of the input signal.*
- This is called *input backoff (IBO)* and results in a proportional *output backoff (OBO).*
- High backoff *reduces the power efficiency of the HPA*, and may *limit the battery life* for mobile applications.



POWER BACKOFF

- Power Backoff in an amplifier is a power level *below the saturation point* at which the amplifier will continue to operate in the linear region even if there is a slight increase in the input power level.
- Usually, *power amplifiers operate close to the saturation point* as that is where efficiency is maximum.
- However, at this point, *a small increase in input power* can push the amplifier from the *linear mode to the saturated mode*.
- Thus to ensure it operates in the linear region we lower the power level from point of maximum efficiency to ensure that it operates in the linear region if there is a slight increase in power.
- ***Amount by which we lower the power level is called Power Backoff.***



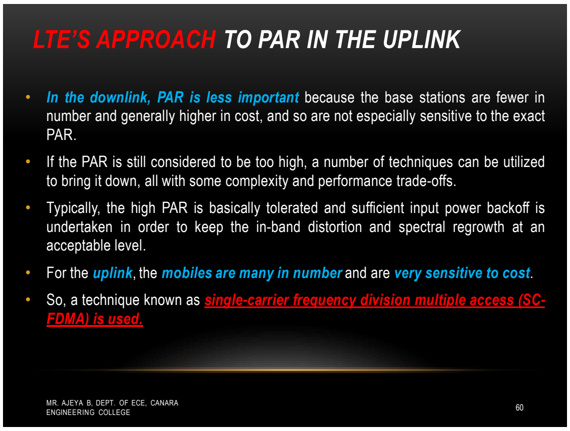


THE PAR PROBLEM…

- In addition to inefficiency in terms of power, the coverage range is reduced and the cost of the HPA is higher than would be mandated by the average power requirements.
- The input backoff is defined as

$$IBO = 10\log_{10}\frac{P_{inSat}}{P_{in}}$$

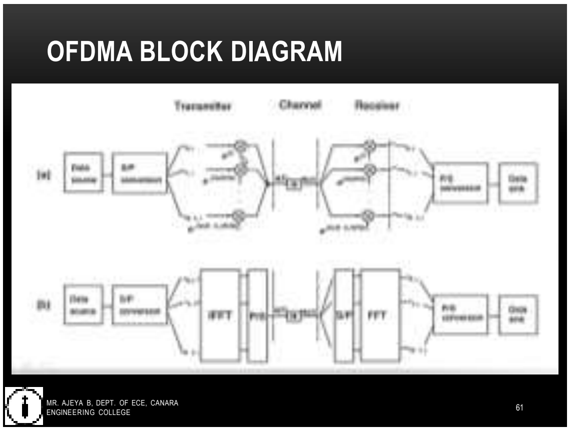
  - where P_{inSat} is the saturation power (above which is the nonlinear region) and P_{in} is the average input power.
  - The amount of *backoff* is usually *greater* than or equal to the *PAR of the signal*.



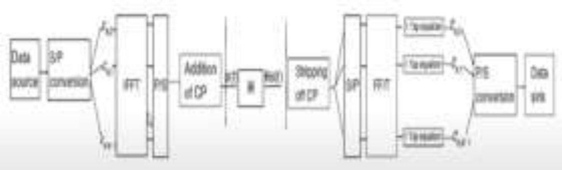
LTE'S APPROACH TO PAR IN THE UPLINK

- *In the downlink, PAR is less important* because the base stations are fewer in number and generally higher in cost, and so are not especially sensitive to the exact PAR.
- If the PAR is still considered to be too high, a number of techniques can be utilized to bring it down, all with some complexity and performance trade-offs.
- Typically, the high PAR is basically tolerated and sufficient input power backoff is undertaken in order to keep the in-band distortion and spectral regrowth at an acceptable level.
- For the *uplink*, the *mobiles are many in number* and are *very sensitive to cost*.
- So, a technique known as *single-carrier frequency division multiple access (SC-FDMA) is used.*

OFDMA BLOCK DIAGRAM



WITH CYCLIC PREFIX

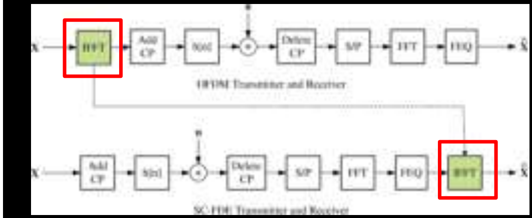


SINGLE-CARRIER FREQUENCY DOMAIN EQUALIZATION (*SC-FDE*)

- An alternative approach to OFDM is the less popular but conceptually similar single-carrier frequency domain equalization (SC-FDE) approach to ISI suppression.
- ***SC-FDE maintains OFDM's three most important benefits:***
  - (1) ***low complexity*** even for severe multipath channels
  - (2) ***excellent BER performance***, close to theoretical bounds; and
  - (3) ***decoupling of ISI from other types of interference***, notably spatial interference, which is very useful when using multiple antenna transmission.
- By utilizing single-carrier transmission, the ***peak-to-average ratio is also reduced significantly*** (by several dB) relative to multicarrier modulation.

SC-FDE SYSTEM DESCRIPTION

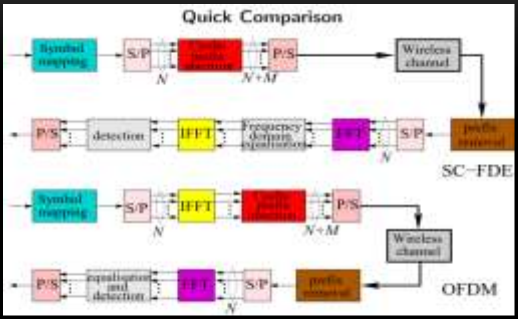
- ***Frequency domain equalization*** is used in both OFDM and SC-FDE systems primarily in order to ***reduce the complexity*** inherent to time-domain equalization.
- we can see that the only apparent ***difference between the two systems*** is that the ***IFFT is moved to the end of the receive chain*** rather than operating at the transmitter, to create a multicarrier waveform as in OFDM.



OFDM Transmitter and Receiver

SC-FDE Transmitter and Receiver

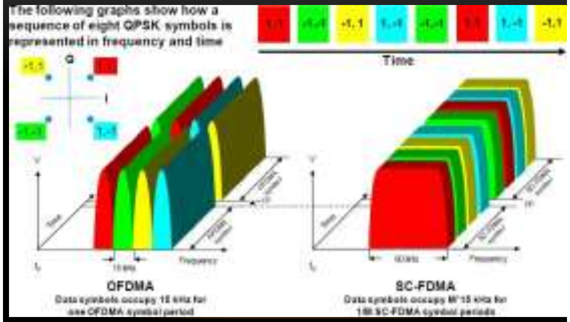
SC-FDE AND OFDM


Quick Comparison

SPOT "DIFFERENCE"

- ***SC-FDE***: data symbols are "time-domain" quantity and transmitted directly
- ***OFDM***: data symbols are "frequency-domain" quantity (and are IDFT into "time-domain" for transmission)

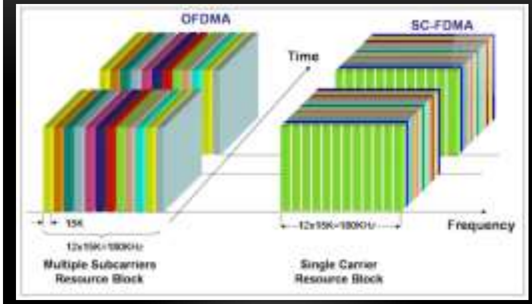
SC-FDE AND OFDM


The following graphs show how a sequence of eight QPSK symbols is represented in frequency and time

OFDMA
Data symbols occupy 15 kHz for one OFDMA symbol period

SC-FDMA
Data symbols occupy M*15 kHz for 1/M SC-FDMA symbol periods

SC-FDE AND OFDM



SC-FDE SYSTEM DESCRIPTION…

- An SC-FDE system still utilizes a cyclic prefix at least as long as the channel delay spread, but now the transmitted signal is simply a sequence of QAM symbols,
  - which have *low PAR, on the order of 4–5 dB* depending on the constellation size.
- Considering that an unmodulated sine wave has a PAR of 3 dB, it is clear that the PAR cannot be lowered much below that of an SC-FDE system.

- PAPR is proportional to square of number of carriers
- Hence SCFDMA has lower PAPR

PAPR OF SINE WAVE

$$PAPR_{dB} = 10\log_{10}\frac{|x_{peak}|^2}{x_{rms}^2} = C_{dB}$$

Wave type	Waveform	RMS value	Crest factor	PAPR (dB)
DC		1	1	0.0 dB
Sine wave		$\frac{1}{\sqrt{2}} \approx 0.707$[6]	$\sqrt{2} \approx 1.414$	3.01 dB
QPSK		1	1	1.761 dB[7]
8PSK				3.3 dB[8]
π/4DQPSK				3.0 dB[8]
OQPSK				3.3 dB[8]
OFDM			4	~12 dB

SC-FDE SYSTEM DESCRIPTION…

- *The action*, in an SC-FDE system, *is at the receiver*.
- As in an OFDM system, an FFT is applied, but in an SC-FDE system this operation *moves the received signal into the frequency domain*.
- Because of the application of the cyclic prefix, the *received signal appears to be circularly convolved*, that is,

$$y[n] = x[n] \circledast h[n] + w[n]$$

where $w[n]$ is noise.

- Therefore, $\mathrm{FFT}\{y[n]\} \triangleq Y[m] = H[m]X[m] + W[m]$

SC-FDE SYSTEM DESCRIPTION…

- Just as in OFDM, with the important distinction being that now the frequency domain version **X[m] is not precisely the data symbols**, but rather the FFT of the data symbols *x*[*n*].
- Analogously, recall that in an OFDM system the ***transmitted time-domain signal x[n] was not the actual data symbols***, but rather the ***IFFT of the actual data symbols***.

SC-FDE SYSTEM DESCRIPTION…

- After the FFT, a simple 1-tap FEQ can be applied that *inverts* each virtual subcarrier, so that

$$\breve{X}[m] = \frac{Y[m]}{H[m]}$$

- The resulting signal can then be converted back into the time domain using an IFFT operation to give x[m], which are estimates of the desired data symbols.
- Naturally, in practice **H[m] must be estimated at the receiver *using pilot signals or other standard methods.***

DESIGN CONSIDERATIONS FOR SC-FDE AND OFDM

- *SC-FDE* has a *lower-complexity transmitter* but a *higher-complexity receiver*, compared to OFDM.
- *Base station* would therefore *perform THREE IFFT/FFT operations*
- *The mobile*, which is more power- and cost-sensitive, would *perform only a single FFT operation* (to receive its OFDM waveform from the base station).
- Adding in SC-FDE's benefits of reduced PAR and the commensurate cost and power savings, it appears that
  - ✓ the case for *using SC-FDE* in the *uplink* of a wideband data system is *favourable* indeed.

DESIGN CONSIDERATIONS FOR SC-FDE AND OFDM…

- Commonly cited *disadvantage* of SC-FDE is that it has a nominally **more dispersive spectrum compared to OFDM**
- Because OFDM has a higher PAR, it is more subject to clipping that can cause spectral dispersion
- The combination of SC-FDE with MIMO is not as natural because detection cannot be done in the frequency domain.

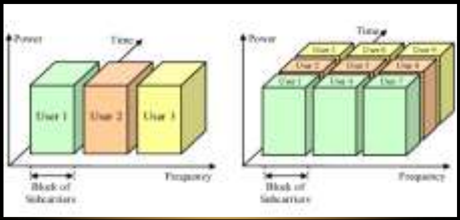
Frequency Domain Multiple Access: OFDMA and SC-FDMA

MULTIPLE ACCESS FOR OFDM SYSTEMS

- OFDM is ***not a multiple-access strategy***, but rather a technique for mitigating frequency selectivity (inter-symbol interference).
- OFDM creates many parallel streams of data that can in principle be used by different users.
- Most previous ***OFDM systems*** such as ***DSL, 802.11a/g***, and ***earlier 802.16/WiMAX*** systems (prior to the 802.16e standard) have used what can be called "*single-user OFDM*"
  - ➢ often simply "OFDM"—***all the subcarriers are used by a single user at a time.***

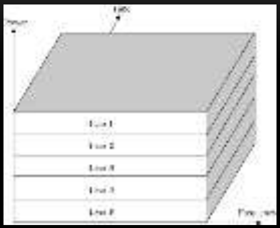
MULTIPLE ACCESS OVERVIEW

- Three Multicarrier based multiple access techniques used for OFDM
- **OFDM-FDMA**
- **OFDM-TDMA**
- **OFDM-CDMA**



FDMA (left) and a combination of FDMA with TDMA (right).

MULTIPLE ACCESS OVERVIEW…



CDMA's users share time and frequency slots but employ codes that allow the users to be separated by the receiver.

ORTHOGONAL FREQUENCY DIVISION MULTIPLE ACCESS (OFDMA)

- OFDMA systems allocate ***subscribers time-frequency slices*** (in LTE, “resource grids”) consisting of *M* subcarriers over some number of consecutive OFDM symbols in time (in LTE, most commonly seven).
- The *M* subcarriers can either be
  1. ***spread out over the band***, often called a “distributed,” “comb,” or “diversity” allocation or
  2. ***bunched together in M contiguous subcarriers***, which is often called a “band AMC,” “localized,” or “grouped” cluster.
- The distributed allocation achieves frequency diversity over the entire band, and would typically rely on interleaving and coding to correct errors caused by poor subcarriers.

OFDMA: HOW IT WORKS

- The basic flow is very ***similar to an OFDM system except*** for now ***K users share the L subcarriers***, with each user being allocated M_k subcarriers.

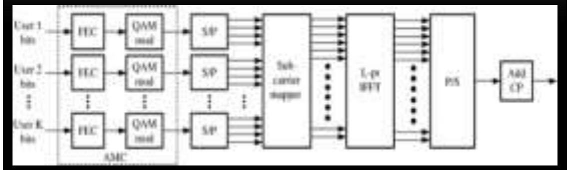


Fig: OFDMA downlink transmitter

OFDMA: HOW IT WORKS…

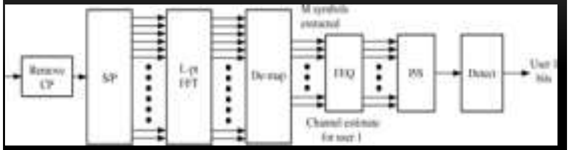
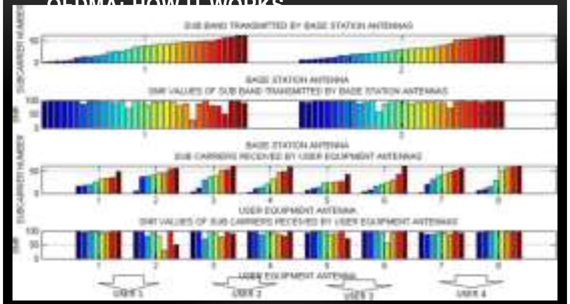


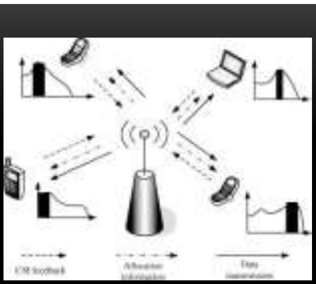
Fig: OFDMA downlink receiver for user 1.

- Each of the *K* active users—who by design have orthogonal subcarrier assignments —have a different receiver that only detects the M_k subcarriers intended for it.

OFDMA: HOW IT WORKS

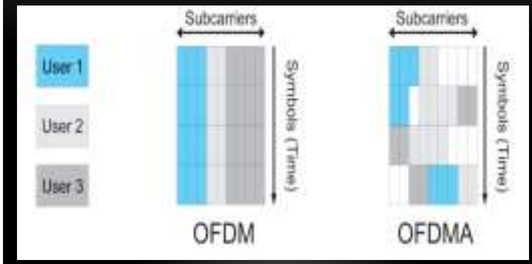


OFDMA



- In OFDMA, the base station allocates each user a fraction of the subcarriers, preferably, in a range where they have a strong channel.

OFDMA VS OFDM



THE ADVANTAGES OF OFDMA

- Robust multipath suppression, relatively low complexity, and the creation of frequency diversity.
- In addition, OFDMA is a flexible multiple access technique that can accommodate many users with widely varying applications, data rates, and QOS requirements.
- Because the multiple access is performed in the digital domain (before the IFFT operation), dynamic, flexible, and efficient bandwidth allocation is possible.
- Lower data rates (such as voice) and bursty data are handled much more efficiently in OFDMA than in single-user OFDM (i.E., OFDM-TDMA) or with CSMA.

SINGLE-CARRIER FREQUENCY DIVISION MULTIPLE ACCESS (*SC-FDMA*)

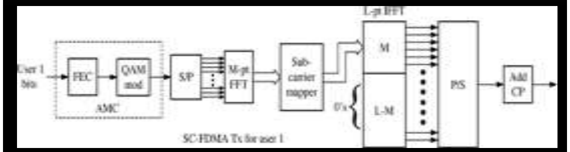
- Employed in the LTE uplink.
- Conceptually, this system evolves naturally from the single-carrier frequency domain equalization (SC-FDE) modulation approach.
- However, SC-FDMA more closely resembles OFDMA because it still requires an IFFT operation at the transmitter in order to separate the users.
- Because SC-FDE is truly a single-carrier modulation technique, it is not possible for an uplink user to use only part of the spectrum, or part of an SC-FDE block.

SC-FDMA

- The goal of SC-FDMA is:
  1. To take the low peak-to-average ratio (PAR) properties of SC-FDE and
  2. Achieve them in an OFDMA-type system that allows partial usage of the frequency band
- In fact, SC-FDMA can reasonably be called "FFT (or DFT) precoded OFDMA"

SC-FDMA: HOW IT WORKS

- very similar to the OFDMA uplink transmitter
- The only difference being that the user's M_k complex symbols are pre-processed with an FFT of size M_k.



SC-FDMA uplink transmitter for user 1, where user 1 is allocated subcarriers 1, 2, ..., *M* of *L* total subcarriers

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SC-FDMA: HOW IT WORKS...

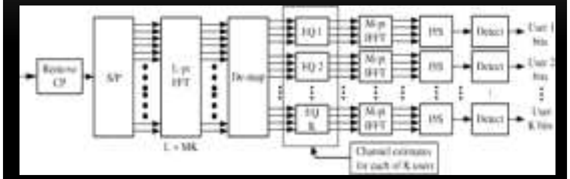


Fig: SC-FDMA uplink receiver.

Much like the OFDMA receiver, here we explicitly assume that each user occupies a fraction *M/L* of the spectrum.

SC-FDMA ADVANTAGES AND DISADVANTAGES

- In SC-FDMA, Key advantage of OFDMA is preserved: ***only part of the frequency spectrum is used by any one user at a time***.
- PAR of SC-FDMA is significantly lower than OFDMA
- The tradeoffs between SC-FDMA and OFDMA are also closely related to the trade-offs SC-FDE faces versus OFDM.
- In particular, ***SC-FDMA can experience more spectral leakage than OFDMA***, and ***achieve frequency diversity differently***, leading to slight differences in performance.
- SC-FDMA has a ***complexity disadvantage*** versus OFDMA in both the transmitter and receiver as an additional FFT of size M_k has to be performed for each user at the transmitter and receiver.

OFDMA AND SC-FDMA IN LTE

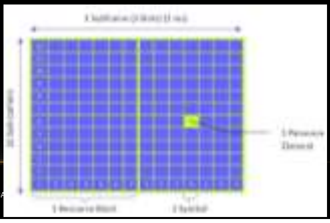
The LTE Time-Frequency Grid

- In LTE, mobile units are allocated groups of subcarriers over time and frequency known as a ***resource block***.
- The size of the resource block is chosen to balance a tradeoff between granularity and overhead.
- A typical resource block consists of ***12 subcarriers over 7 OFDM symbols***, also referred to as a timeslot.
- A ***timeslot in LTE spans 0.5 msec*** and ***two consecutive timeslots create a subframe***.

OFDMA AND SC-FDMA IN LTE….

The LTE Time-Frequency Grid…

- Resources are allocated to users in units of resource blocks over a subframe
  - ➢ 12 subcarriers over 2 × 7 = 14 OFDM symbols for a total of 168 "resource elements," which in practice are QAM symbols.
- Not all the 168 resource elements can be used for data since some are used for various layer 1 and layer 2 control messages.



THE LTE TIME-FREQUENCY GRID….

- The subcarriers of a resource block can be allocated in ***one of two ways***.

1. ***Distributed subcarrier allocation***:
   - Takes advantage of frequency diversity by spreading the resource block hop across the entire channel bandwidth.
   - This can be accomplished by using a "comb" pattern at any given point of time for a given user, so that its subcarriers occur at even intervals across the entire frequency bandwidth.
   - This approach is typically used in the downlink (OFDMA) when distributed subcarrier allocation is used.

THE LTE TIME-FREQUENCY GRID…

2. ***Adjacent subcarrier allocation.***

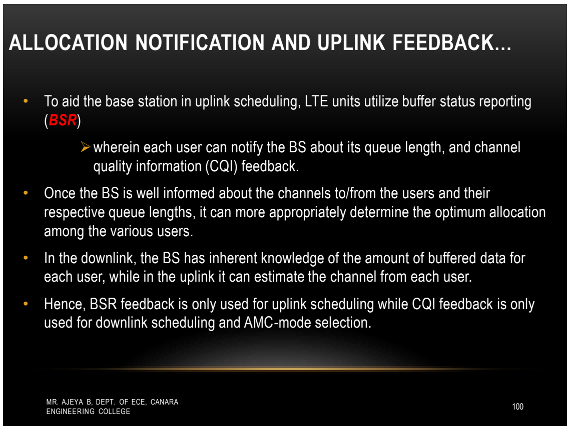
- This approach relies on a channel-aware allocation of resources,
  - ➢ so that each user can be allocated a resource block where they have a strong channel.
- Since a block of 12 subcarriers is typically smaller than the coherence bandwidth of the channel, frequency diversity is not achieved
  - ➢ which is helpful as long as the scheduler is able to assign "good" blocks to each user.

ALLOCATION NOTIFICATION AND UPLINK FEEDBACK

- In order ***for each MS to know*** **which subcarriers to use in downlink reception and uplink transmission**, the ***BS must broadcast this information to the pool of active users in its cell***.
- Similar to previous UMTS standards such as wideband CDMA, overhead signalling is done on a logical control channel, in this case, the PDCCH (physical downlink control channel).
- The PDCCH specifies the following:
  - i. • Downlink resource block allocation
  - ii. • Uplink resource block allocation
  - iii. • QAM constellation to use per resource block
  - iv. • Type and rate of coding to use per resource block

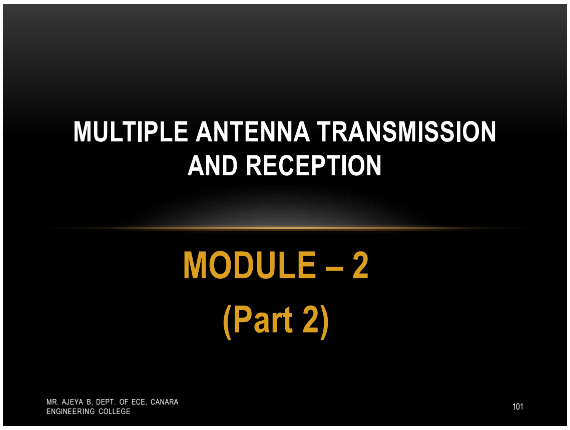
ALLOCATION NOTIFICATION AND UPLINK FEEDBACK…

- Once a user is able to **decode the PDCCH**, **it knows precisely where to receive** (**downlink**) or to transmit (**uplink**), and **how**.
- The PDCCH is sent over the first 2–3 OFDM symbols of each subframe across all the subcarriers.
- Recall that each allocation, which consists of a resource block subframe, consisted of 168 subcarriers over 14 OFDM sybmols.
- Since the ***first 2–3 symbols in each subframe are used by the PDCCH***, about ***14–21% of the total downlink capacity is used by the PDCCH***.



ALLOCATION NOTIFICATION AND UPLINK FEEDBACK…

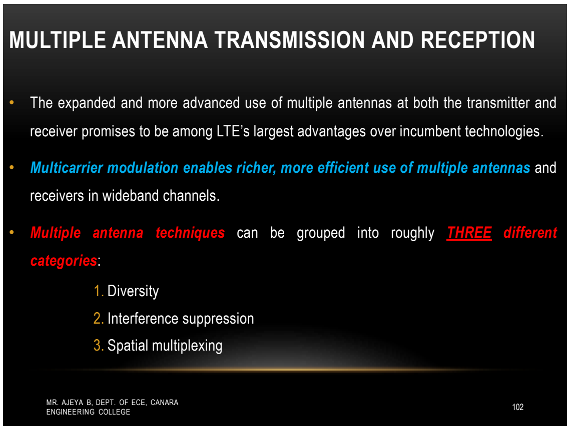
- To aid the base station in uplink scheduling, LTE units utilize buffer status reporting (*BSR*)
  - ➢wherein each user can notify the BS about its queue length, and channel quality information (CQI) feedback.
- Once the BS is well informed about the channels to/from the users and their respective queue lengths, it can more appropriately determine the optimum allocation among the various users.
- In the downlink, the BS has inherent knowledge of the amount of buffered data for each user, while in the uplink it can estimate the channel from each user.
- Hence, BSR feedback is only used for uplink scheduling while CQI feedback is only used for downlink scheduling and AMC-mode selection.



MULTIPLE ANTENNA TRANSMISSION AND RECEPTION

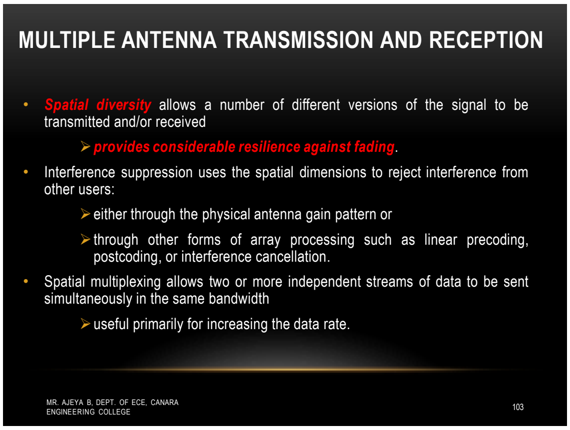
MODULE – 2

(Part 2)



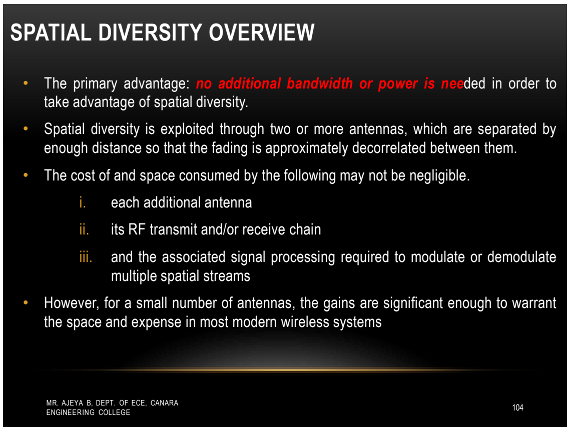
MULTIPLE ANTENNA TRANSMISSION AND RECEPTION

- The expanded and more advanced use of multiple antennas at both the transmitter and receiver promises to be among LTE's largest advantages over incumbent technologies.
- ***Multicarrier modulation enables richer, more efficient use of multiple antennas*** and receivers in wideband channels.
- ***Multiple antenna techniques*** can be grouped into roughly ***THREE different categories***:
  1. Diversity
  2. Interference suppression
  3. Spatial multiplexing



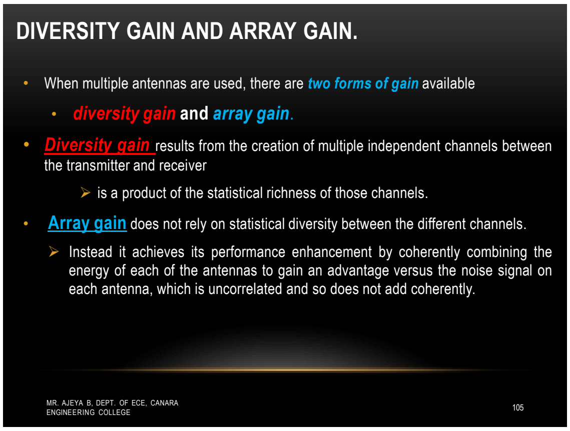
MULTIPLE ANTENNA TRANSMISSION AND RECEPTION

- *Spatial diversity* allows a number of different versions of the signal to be transmitted and/or received
  - ➢***provides considerable resilience against fading***.
- Interference suppression uses the spatial dimensions to reject interference from other users:
  - ➢either through the physical antenna gain pattern or
  - ➢through other forms of array processing such as linear precoding, postcoding, or interference cancellation.
- Spatial multiplexing allows two or more independent streams of data to be sent simultaneously in the same bandwidth
  - ➢useful primarily for increasing the data rate.



SPATIAL DIVERSITY OVERVIEW

- The primary advantage: *no additional bandwidth or power is needed* in order to take advantage of spatial diversity.
- Spatial diversity is exploited through two or more antennas, which are separated by enough distance so that the fading is approximately decorrelated between them.
- The cost of and space consumed by the following may not be negligible.
  - i. each additional antenna
  - ii. its RF transmit and/or receive chain
  - iii. and the associated signal processing required to modulate or demodulate multiple spatial streams
- However, for a small number of antennas, the gains are significant enough to warrant the space and expense in most modern wireless systems



DIVERSITY GAIN AND ARRAY GAIN.

- When multiple antennas are used, there are ***two forms of gain*** available
  - ***diversity gain*** **and** ***array gain***.
- ***Diversity gain*** results from the creation of multiple independent channels between the transmitter and receiver
  - ➢ is a product of the statistical richness of those channels.
- **Array gain** does not rely on statistical diversity between the different channels.
  - ➢ Instead it achieves its performance enhancement by coherently combining the energy of each of the antennas to gain an advantage versus the noise signal on each antenna, which is uncorrelated and so does not add coherently.

INCREASING THE DATA RATE WITH SPATIAL DIVERSITY

- Receive diversity techniques also increase the average received SNR at best linearly due to the array gain.
- The Shannon capacity formula gives the maximum achievable data rate of a single communication link in additive white Gaussian noise (AWGN) as:

$$C = B\log_2(1+\gamma),$$

  - where *C* is the “capacity,” or maximum error-free data rate, *B* is the bandwidth of the channel, and *γ* is again the SNR (or SINR).
- Due to advances in coding, and with sufficient diversity, it may be possible to approach the Shannon limit in some wireless channels.

INCREASED COVERAGE OR REDUCED TRANSMIT POWER

- The benefits of diversity can also be harnessed to increase the coverage area
- and to reduce the required transmit power
  - although these gains directly compete with each other
  - as well as with the achievable reliability and data rate.

RECEIVE DIVERSITY

- The most prevalent form of spatial diversity is receive diversity
  - ➢ often with just two antennas.
- This type of diversity is nearly ubiquitous—N_r = 2 being by far the most common—on cellular base stations and wireless LAN access points
  - ✓ will be mandatory for LTE base stations and handsets.
- Receive diversity on its own places no particular requirements on the transmitter
  - ✓ but requires a receiver that processes the N_r received streams and combines them in some fashion.
- Because receive diversity places no requirements on the transmitter, these techniques are not specified in the LTE standard.

RECEIVE DIVERSITY…

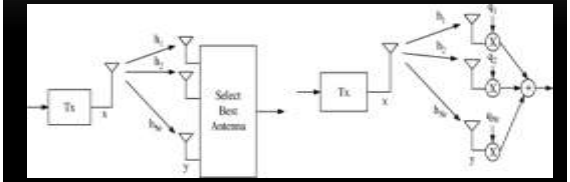


Figure : Receive diversity: selection combining (left) and maximal ratio combining (right).

SELECTION COMBINING

- Simplest type of “***combiner***,” in that it simply ***estimates the instantaneous strengths of each of the N_r streams***, and **selects the highest one**.
- Since SC ignores the useful energy on the other streams, it is clearly suboptimal, but its simplicity and reduced hardware and power requirements make it attractive for narrowband channels.
- In a wideband channel, different coherence bands will have different SNRs, and so although selection diversity can be used on each band:
  - ➢ that would likely require all antennas to be active for at least one band
  - ➢ which nullifies one of the main arguments in favor of selection diversity.
- If all the antennas and RF chains have to be active, it is usually better to use MRC.

SELECTION COMBINING…

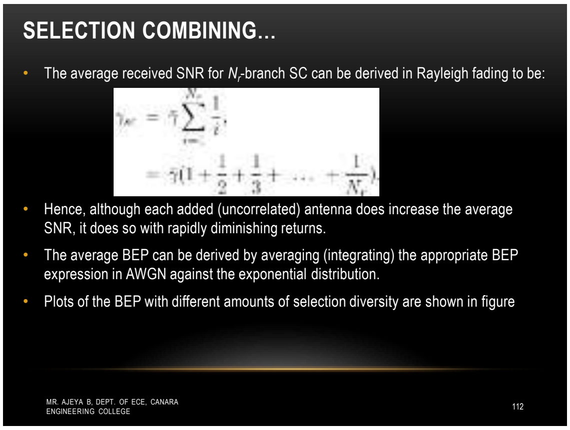
- The diversity gain from employing selection combining can be confirmed quite quickly by considering the outage probability, defined as the probability that the received SNR drops below some required threshold, $P_{out} = P[\gamma < \gamma_o] = p$.
- Assuming N_r uncorrelated receptions of the signal

$$\begin{aligned} P_{out} &= P[\gamma_1 < \gamma_o, \gamma_2 < \gamma_o, \ldots, \gamma_{N_r} < \gamma_o], \\ &= P[\gamma_1 < \gamma_o]P[\gamma_2 < \gamma_o]\ldots P[\gamma_{N_r} < \gamma_o], \\ &= p^{N_r}. \end{aligned}$$

For a Rayleigh fading channel: $p = 1 - e^{-\gamma_o/\bar{\gamma}}$

- Where γ is the average received SNR at that location (for example, due to path loss).
- Thus, selection combining decreases the outage probability to:

$$P_{SC} = [1 - e^{-\gamma_o/\bar{\gamma}}]^{N_r}$$

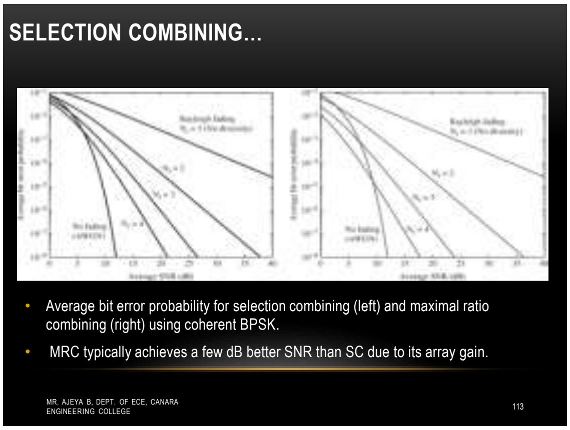


SELECTION COMBINING…

- The average received SNR for N_r-branch SC can be derived in Rayleigh fading to be:

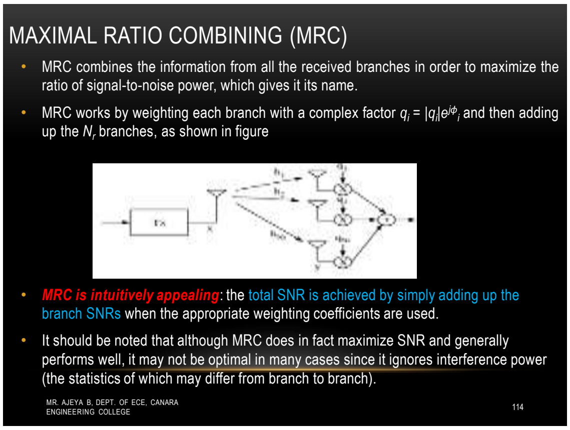
$$\bar{\gamma}_{\Sigma} = \bar{\gamma}\sum_{i=1}^{N_r}\frac{1}{i},$$
$$= \bar{\gamma}\left(1+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{N_r}\right).$$

- Hence, although each added (uncorrelated) antenna does increase the average SNR, it does so with rapidly diminishing returns.
- The average BEP can be derived by averaging (integrating) the appropriate BEP expression in AWGN against the exponential distribution.
- Plots of the BEP with different amounts of selection diversity are shown in figure



SELECTION COMBINING…

- Average bit error probability for selection combining (left) and maximal ratio combining (right) using coherent BPSK.
- MRC typically achieves a few dB better SNR than SC due to its array gain.



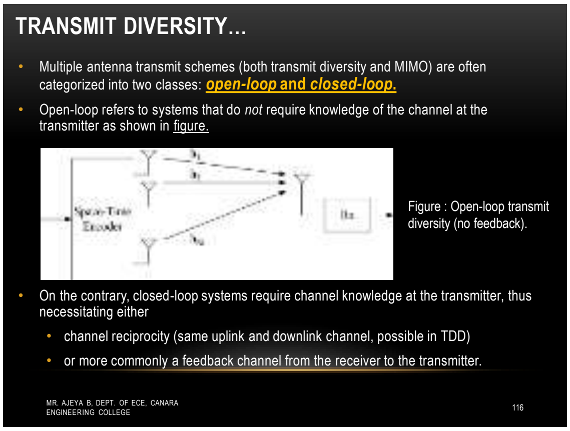
MAXIMAL RATIO COMBINING (MRC)

- MRC combines the information from all the received branches in order to maximize the ratio of signal-to-noise power, which gives it its name.
- MRC works by weighting each branch with a complex factor $q_i = |q_i|e^{j\phi_i}$ and then adding up the N_r branches, as shown in figure

- ***MRC is intuitively appealing***: the total SNR is achieved by simply adding up the branch SNRs when the appropriate weighting coefficients are used.
- It should be noted that although MRC does in fact maximize SNR and generally performs well, it may not be optimal in many cases since it ignores interference power (the statistics of which may differ from branch to branch).

TRANSMIT DIVERSITY

- Spatial transmit diversity is a more recent development than spatial receive diversity, and has become widely understood and implemented only in the early 2000s.
- Because the signals sent from different transmit antennas interfere with one another at the receiver, additional signal processing is required at both the transmitter and receiver in order to achieve diversity while removing or at least attenuating the spatial interference.
- Transmit diversity is particularly useful in the downlink since the base station can usually accommodate more antennas than the mobile station.
- Additionally, if multiple antennas are already present at the base station for uplink receive diversity, the incremental cost of using them for transmit diversity is small.



TRANSMIT DIVERSITY…

- Multiple antenna transmit schemes (both transmit diversity and MIMO) are often categorized into two classes: ***open-loop and closed-loop.***
- Open-loop refers to systems that do *not* require knowledge of the channel at the transmitter as shown in figure.

Figure : Open-loop transmit diversity (no feedback).

- On the contrary, closed-loop systems require channel knowledge at the transmitter, thus necessitating either
  - channel reciprocity (same uplink and downlink channel, possible in TDD)
  - or more commonly a feedback channel from the receiver to the transmitter.

OPEN-LOOP TRANSMIT DIVERSITY: 2 × 1 SPACE-FREQUENCY BLOCK CODING

- The most popular open-loop transmit diversity scheme is space-time (in LTE, space-frequency) coding
  - ➢ where a particular code known to the receiver is applied at the transmitter.
- The receiver must know the channel to decode a space-time code

OPEN-LOOP TRANSMIT DIVERSITY: 2 × 1 SPACE-FREQUENCY BLOCK CODING

Siavash Alamouti

- A key breakthrough in the late 1990s was a space-time block code (STBC) referred to as either the ***Alamouti code*** or the orthogonal space-time block code (OSTBC).
- This simple code has become the most popular means of achieving transmit diversity due to
    - ✓ its ease of implementation (linear at both the transmitter and receiver), and
    - ✓ its optimality with regards to diversity order.

OPEN-LOOP TRANSMIT DIVERSITY: 2 × 1 SPACE-FREQUENCY BLOCK CODING…

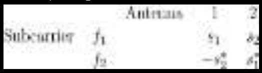
- Conceived for a narrowband fading channel,
    - ➢ STBCs can easily be adapted to a wideband fading channel using OFDM by utilizing adjacent subcarriers rather than consecutive symbols.
- Mathematically and conceptually, there is no difference between SFBCs and the more common STBCs:
    - instead of adjacent subcarriers as denoted below, STBCs use consecutive symbols in time.

OPEN-LOOP TRANSMIT DIVERSITY: 2 × 1 SPACE-FREQUENCY BLOCK CODING…

- ***SFBCs are preferred*** to STBCs because they ***experience less delay*** and are ***less likely to suffer from channel variations***.
- STBCs would require two OFDM symbols to be encoded (and decoded) over
    - ➢ which significantly increases delay while also increasing the likelihood of channel variation over the code block,
    - ➢ which as we will see is contrary to the standard decoding model.

2 × 1 ALAMOUTI SFBC

- The simplest SFBC corresponds to two transmit antennas and a single receive antenna.
- If two symbols to be transmitted are s_1 and s_2, the Alamouti code sends the following over two subcarriers f_1 and f_2:



	Antennas	1	2
Subcarrier	f_1	s_1	s_2
	f_2	$-s_2^*$	s_1^*

- The 2 × 1 Alamouti SFBC is referred to as a rate 1 code, since the data rate is neither increased nor decreased; two symbols are sent over two adjacent subcarriers.
- Rather than directly increasing the data rate, the ***goal*** of space-frequency block coding is to ***harness the spatial diversity of the channel.***

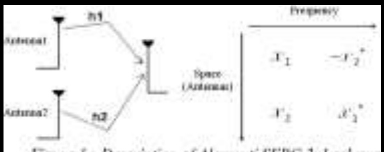
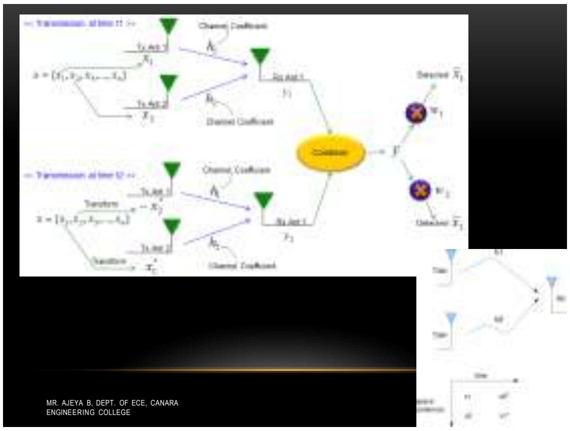
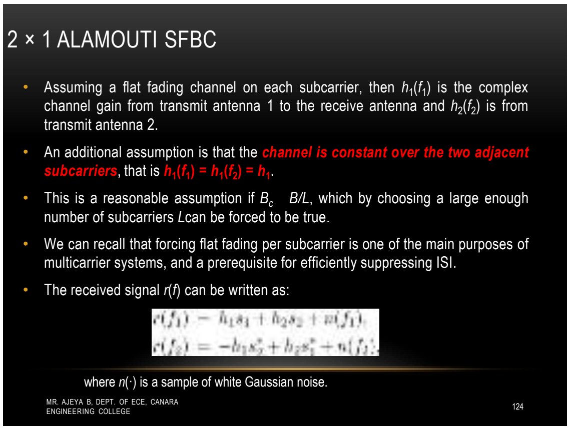


Figure 1 – Description of Alamouti SFBC 2x1 scheme



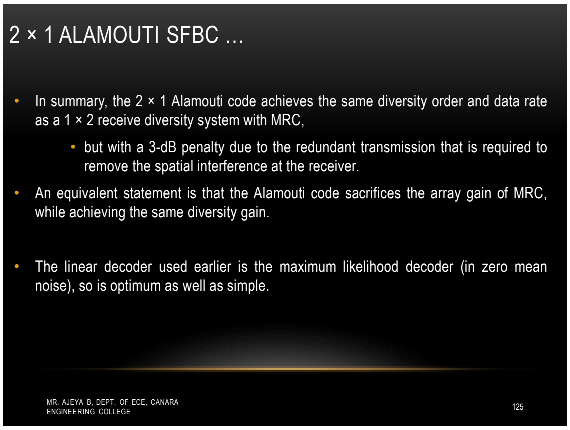


2 × 1 ALAMOUTI SFBC

- Assuming a flat fading channel on each subcarrier, then $h_1(f_1)$ is the complex channel gain from transmit antenna 1 to the receive antenna and $h_2(f_2)$ is from transmit antenna 2.
- An additional assumption is that the ***channel is constant over the two adjacent subcarriers***, that is $h_1(f_1) = h_1(f_2) = h_1$.
- This is a reasonable assumption if B_c B/L, which by choosing a large enough number of subcarriers Lcan be forced to be true.
- We can recall that forcing flat fading per subcarrier is one of the main purposes of multicarrier systems, and a prerequisite for efficiently suppressing ISI.
- The received signal $r(f)$ can be written as:

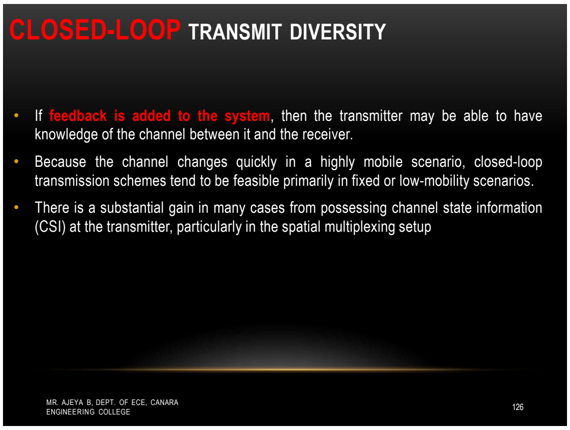
$$
\begin{aligned}
r(f_1) &= h_1 s_1 + h_2 s_2 + n(f_1)\\
r(f_2) &= -h_1 s_2^* + h_2 s_1^* + n(f_2)
\end{aligned}
$$

where $n(\cdot)$ is a sample of white Gaussian noise.



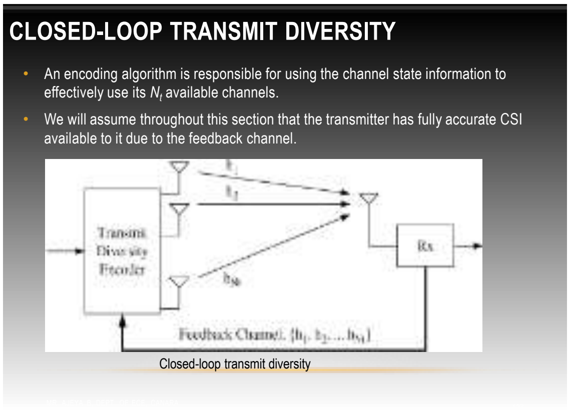
2 × 1 ALAMOUTI SFBC …

- In summary, the 2 × 1 Alamouti code achieves the same diversity order and data rate as a 1 × 2 receive diversity system with MRC,
  - but with a 3-dB penalty due to the redundant transmission that is required to remove the spatial interference at the receiver.
- An equivalent statement is that the Alamouti code sacrifices the array gain of MRC, while achieving the same diversity gain.
- The linear decoder used earlier is the maximum likelihood decoder (in zero mean noise), so is optimum as well as simple.



CLOSED-LOOP TRANSMIT DIVERSITY

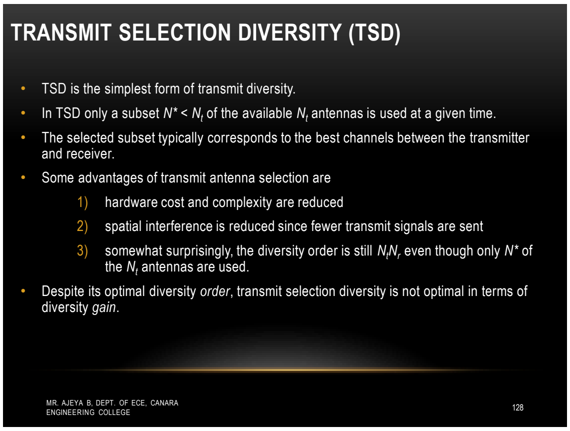
- If **feedback is added to the system**, then the transmitter may be able to have knowledge of the channel between it and the receiver.
- Because the channel changes quickly in a highly mobile scenario, closed-loop transmission schemes tend to be feasible primarily in fixed or low-mobility scenarios.
- There is a substantial gain in many cases from possessing channel state information (CSI) at the transmitter, particularly in the spatial multiplexing setup



CLOSED-LOOP TRANSMIT DIVERSITY

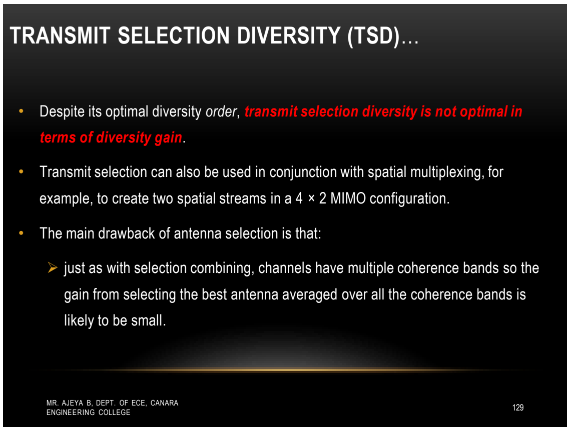
- An encoding algorithm is responsible for using the channel state information to effectively use its N_t available channels.
- We will assume throughout this section that the transmitter has fully accurate CSI available to it due to the feedback channel.

Closed-loop transmit diversity



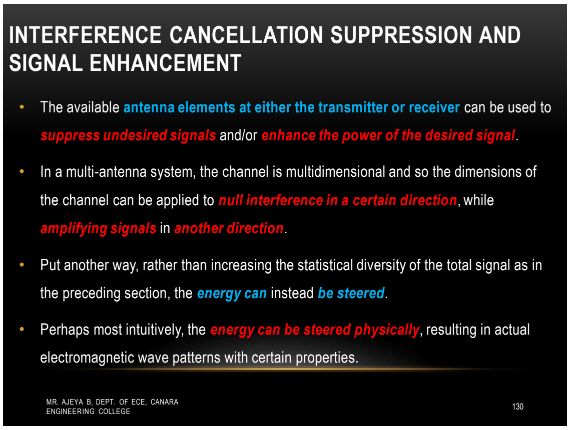
TRANSMIT SELECTION DIVERSITY (TSD)

- TSD is the simplest form of transmit diversity.
- In TSD only a subset $N^* < N_t$ of the available N_t antennas is used at a given time.
- The selected subset typically corresponds to the best channels between the transmitter and receiver.
- Some advantages of transmit antenna selection are
  1) hardware cost and complexity are reduced
  2) spatial interference is reduced since fewer transmit signals are sent
  3) somewhat surprisingly, the diversity order is still $N_t N_r$ even though only N^* of the N_t antennas are used.
- Despite its optimal diversity *order*, transmit selection diversity is not optimal in terms of diversity *gain*.



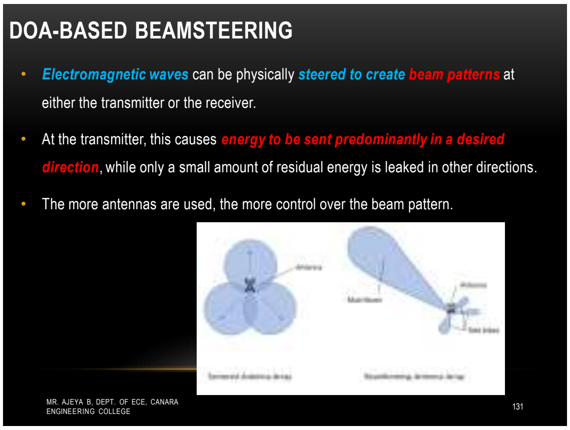
TRANSMIT SELECTION DIVERSITY (TSD)…

- Despite its optimal diversity *order*, ***transmit selection diversity is not optimal in terms of diversity gain***.
- Transmit selection can also be used in conjunction with spatial multiplexing, for example, to create two spatial streams in a 4 × 2 MIMO configuration.
- The main drawback of antenna selection is that:
  - ➢ just as with selection combining, channels have multiple coherence bands so the gain from selecting the best antenna averaged over all the coherence bands is likely to be small.



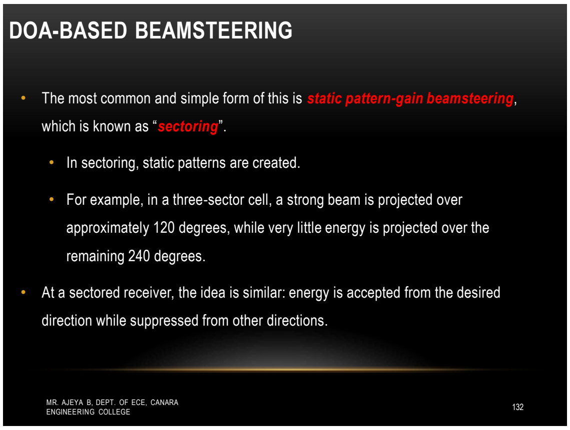
INTERFERENCE CANCELLATION SUPPRESSION AND SIGNAL ENHANCEMENT

- The available **antenna elements at either the transmitter or receiver** can be used to ***suppress undesired signals*** and/or ***enhance the power of the desired signal***.
- In a multi-antenna system, the channel is multidimensional and so the dimensions of the channel can be applied to ***null interference in a certain direction***, while ***amplifying signals*** in ***another direction***.
- Put another way, rather than increasing the statistical diversity of the total signal as in the preceding section, the ***energy can*** instead ***be steered***.
- Perhaps most intuitively, the ***energy can be steered physically***, resulting in actual electromagnetic wave patterns with certain properties.



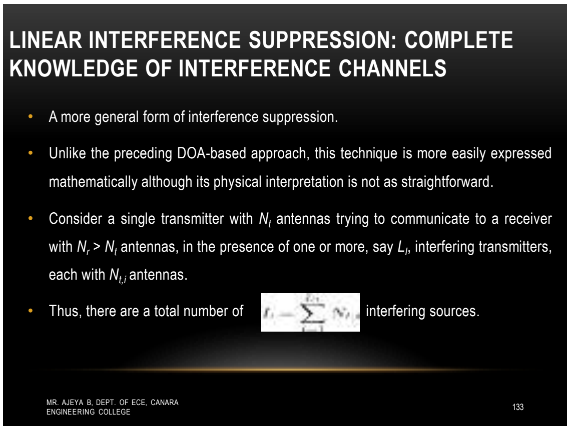
DOA-BASED BEAMSTEERING

- ***Electromagnetic waves*** can be physically ***steered to create beam patterns*** at either the transmitter or the receiver.
- At the transmitter, this causes ***energy to be sent predominantly in a desired direction***, while only a small amount of residual energy is leaked in other directions.
- The more antennas are used, the more control over the beam pattern.



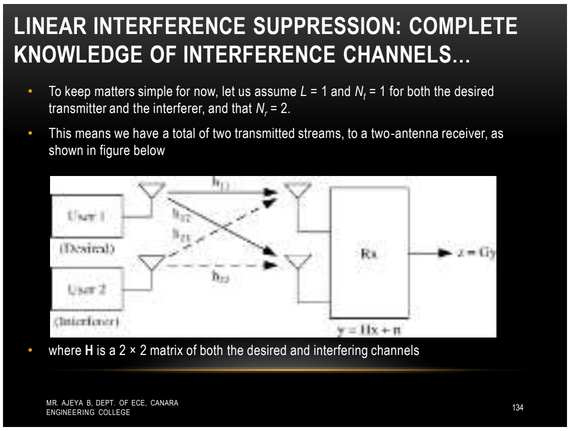
DOA-BASED BEAMSTEERING

- The most common and simple form of this is ***static pattern-gain beamsteering***, which is known as "***sectoring***".
  - In sectoring, static patterns are created.
  - For example, in a three-sector cell, a strong beam is projected over approximately 120 degrees, while very little energy is projected over the remaining 240 degrees.
- At a sectored receiver, the idea is similar: energy is accepted from the desired direction while suppressed from other directions.



LINEAR INTERFERENCE SUPPRESSION: COMPLETE KNOWLEDGE OF INTERFERENCE CHANNELS

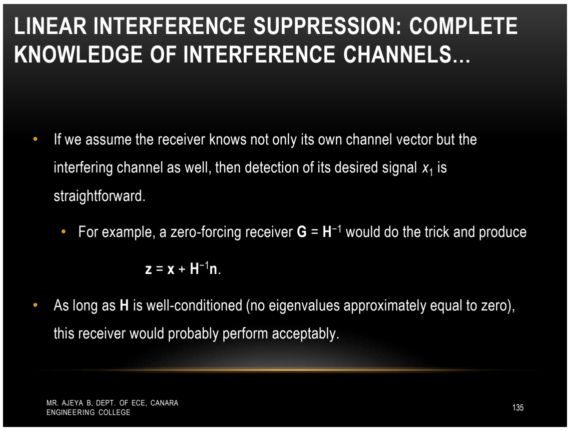
- A more general form of interference suppression.
- Unlike the preceding DOA-based approach, this technique is more easily expressed mathematically although its physical interpretation is not as straightforward.
- Consider a single transmitter with N_t antennas trying to communicate to a receiver with $N_r > N_t$ antennas, in the presence of one or more, say L_I, interfering transmitters, each with $N_{t,i}$ antennas.
- Thus, there are a total number of $N_I = \sum_{i=1}^{L_I} N_{t,i}$ interfering sources.



LINEAR INTERFERENCE SUPPRESSION: COMPLETE KNOWLEDGE OF INTERFERENCE CHANNELS...

- To keep matters simple for now, let us assume $L = 1$ and $N_t = 1$ for both the desired transmitter and the interferer, and that $N_r = 2$.
- This means we have a total of two transmitted streams, to a two-antenna receiver, as shown in figure below

- where **H** is a 2 × 2 matrix of both the desired and interfering channels



LINEAR INTERFERENCE SUPPRESSION: COMPLETE KNOWLEDGE OF INTERFERENCE CHANNELS...

- If we assume the receiver knows not only its own channel vector but the interfering channel as well, then detection of its desired signal x_1 is straightforward.
  - For example, a zero-forcing receiver $\mathbf{G} = \mathbf{H}^{-1}$ would do the trick and produce

$$\mathbf{z} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}.$$

- As long as **H** is well-conditioned (no eigenvalues approximately equal to zero), this receiver would probably perform acceptably.

SPATIAL MULTIPLEXING

- Spatial multiplexing refers to breaking the incoming high rate data stream into *M* parallel data streams, for $M = N_t$ and $N_t \le N_r$.
- Assuming that the streams can be successfully decoded, the spectral efficiency is increased by a factor of *M*.
- This is certainly exciting: it implies that adding antenna elements can greatly increase the data rate without any increase in bandwidth.

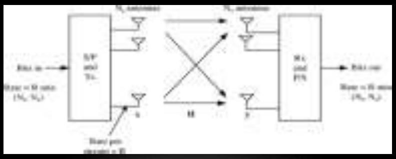


Fig: A spatial multiplexing MIMO system transmits multiple substreams to increase the data rate.

SPATIAL MULTIPLEXING

- Spatial multiplexing has proven challenging in practice
  - largely because this data rate increase comes at the expense of the diversity and/or interference suppression capabilities.

AN INTRODUCTION TO SPATIAL MULTIPLEXING

- The standard mathematical model for spatial multiplexing is very ***similar to*** what was used for ***linear precoding and interference suppression***, i.e.: Y = Hx+n
  - Where the size of the received vector **y** is $N_r \times 1$, the channel matrix **H** is $N_r \times N_t$, the transmit vector **x** is $N_t \times 1$, and the noise **n** is $N_r \times 1$.
- Typically, the transmit vector is normalized by N_t so that each symbol in **x** has average energy ε_x/N_t.
- This keeps the total transmit energy constant with the SISO case for comparison.

AN INTRODUCTION TO SPATIAL MULTIPLEXING

- The channel matrix in particular is of the form:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r1} & h_{N_r2} & \cdots & h_{N_rN_t} \end{bmatrix}$$

- It is usually assumed that the entries in the channel matrix and the noise vector are ***complex Gaussian with zero mean*** and ***diagonal covariance matrices***

AN INTRODUCTION TO SPATIAL MULTIPLEXING

The ***key points*** regarding this single-user MIMO system model are

1. The capacity, or maximum data rate, grows as min(Nt, Nr) log(1 + SNR) when the SNR is large.
   - ✓ When the SNR is high, spatial multiplexing is optimal.
2. When the SNR is low, the capacity-maximizing strategy is to send a single stream of data using diversity precoding.
3. Both of these cases are superior in terms of capacity to space-time coding, where the data rate grows at best logarithmically with Nr.
4. The average SNR of all Nt streams can be maintained without increasing the total transmit power relative to a SISO system
   - However, even a single low eigenvalue in the channel matrix can dominate the error performance.

OPEN-LOOP MIMO: SPATIAL MULTIPLEXING WITHOUT CHANNEL FEEDBACK

- As with multiantenna diversity techniques, ***spatial multiplexing can be performed with or without channel knowledge at the transmitter***.
- The open-loop techniques for spatial multiplexing attempt to suppress the interference that results from all Nt streams being received by each of the Nr antennas.

Table: The similarity of interference suppression techniques for different applications. Complexity is decreasing from left to right.

	Optimum	Interference Cancellation	Linear
Equalization (ISI)	Maximum Likelihood Sequence Detection (MLSE)	Decision feedback equalization	Zero-forcing, MMSE
Multiuser Detection	Optimum MUD	Successive/parallel interference cancellation, iterative MUD	Decorrelating, MMSE
Spatial Multiplexing Receivers	ML detector, sphere decoder (near-optimum)	BLAST	Zero-forcing, MMSE

OPTIMUM DECODING: MAXIMUM LIKELIHOOD DETECTION

- If the channel is unknown at the transmitter, the optimum decoder is the maximum-likelihood decoder, which finds the most likely input vector via a minimum distance criterion: $\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$
- Unfortunately, there is no simple way to compute this, and an exhaustive search must be done over all MNt possible input vectors
  - where M is the order of the modulation (e.g., M = 4 for QPSK).
- The computational complexity is prohibitive for even a small number of antennas.

LINEAR DETECTORS

- As in other situations where the optimum decoder is an intolerably complex maximum likelihood detector, a sensible next step is to consider linear detectors that are capable of recovering the transmitted vector **x**.
- The most obvious such detector is the *zero-forcing detector* that sets the receiver equal to the inverse of the channel $\mathbf{G}_{zf} = \mathbf{H}^{-1}$ when $N_t = N_r$, or more generally to the pseudoinverse: $\mathbf{G}_{zf} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*$

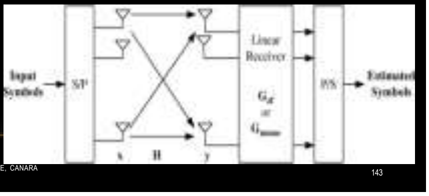


Figure: Spatial multiplexing with a linear receiver.

LINEAR DETECTORS: ZERO-FORCING DETECTOR

- As the name implies, the zero-forcing detector completely removes the spatial interference from the transmitted signal, giving an estimated received vector:

$$\hat{\mathbf{x}} = \mathbf{G}_{zf}\mathbf{y} = \mathbf{G}_{zf}\mathbf{H}\mathbf{x} + \mathbf{G}_{zf}\mathbf{n} = \mathbf{x} + (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{n}$$

- Because $\mathbf{G}_{zf}$ inverts the eigenvalues of **H**, the bad spatial subchannels can severely amplify the noise in **n**.
- This is particularly problematic in interference-limited MIMO systems, and will result in extremely poor performance.
- The zero-forcing detector is therefore ***not practical for LTE.***

LINEAR DETECTORS: MMSE RECEIVER

- A logical ***alternative to*** the ***zero-forcing receiver*** is the Minimum mean square error (***MMSE***) ***receiver***,
  - which attempts to strike a balance between spatial interference suppression and noise enhancement by simply ***minimizing the distortion***.

  Therefore: $\mathbf{G}_{mmse} = \arg\min_{\mathbf{G}} E\|\mathbf{G}\mathbf{y} - \mathbf{x}\|^2$
- which can be derived using the well-known orthogonality principle as:

$$\mathbf{G}_{mmse} = \left(\mathbf{H}^*\mathbf{H} + \frac{\sigma^2}{P_t}\mathbf{I}\right)^{-1}\mathbf{H}^*$$

  - where P_t is the transmitted power.
- In other words, ***as the SNR grows large, the MMSE detector converges to the ZF detector***
  - but at low SNR it prevents the worst eigenvalues from being inverted.

BLAST

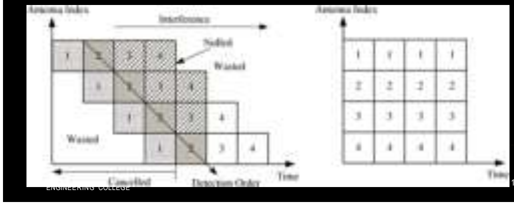
- The earliest known spatial multiplexing receiver was invented and prototyped in Bell Labs and is called **Bell labs LAyered Space-Time (BLAST).**
- Like other spatial multiplexing MIMO systems, BLAST ***consists of parallel "layers" supporting multiple simultaneous data streams***.
- The *layers* (substreams) in BLAST are ***separated by interference cancellation techniques that decouple the overlapping data streams***.
- The two most important techniques are
  1. *diagonal BLAST* (D-BLAST)
  2. *vertical BLAST* (V-BLAST)

D-BLAST

- ***Groups the transmitted symbols into "layers,"*** which are then ***coded in time independently of the other layers***.
- These layers are then cycled to the different transmit antennas in a cyclical manner
  - Resulting in each layer being transmitted in a diagonal of space and time.
- In this way, each symbol stream achieves diversity in time via coding, and in space since it rotates among all the different antennas.
- Therefore, the N_t transmitted streams will equally share the good and bad spatial channels, as well as their priority in the decoding process
- The ***key to the BLAST techniques*** lies in the ***detection of the overlapping*** and ***mutually interfering spatial streams***.
- The diagonal layered structure of D-BLAST can be detected by decoding one layer at a time.

D-BLAST…

- The decoding process for the second of four layers is shown in the left side of figure.
- Each layer is detected by ***nulling*** the layers that have not yet been detected, and ***cancelling*** the layers that have already been detected.
- the layer to the left of the layer-2 block has already been detected and hence subtracted (cancelled) from the received signal while those to the right remain as interference but can be nulled using knowledge of the channel.



D-BLAST…

- The time-domain coding helps compensate for errors or imperfections in the cancellation and nulling process.
- ***Two drawbacks of D-BLAST***
  1. The decoding process is iterative and somewhat complex
  2. The diagonal layering structure wastes space-time slots at the beginning and end of a D-BLAST block.

V-BLAST

- V-BLAST was subsequently addressed in order to reduce the inefficiency and complexity of D-BLAST.
- V-BLAST is ***actually conceptually*** somewhat ***simpler than D-BLAST***.
- In V-BLAST, ***each antenna simply transmits an independent symbol stream*** (for example, QAM symbols).
- A variety of techniques can be used at the receiver to separate the various symbol streams from each other, including several of the techniques discussed elsewhere in this chapter.
- These include linear receivers such as the ZF and MMSE, which take the form at each receive antenna of a length N_r vector that can be used to null out the contributions from the $N_t - 1$ interfering data streams.
- In this case, the post-detection SNR for the *i*th stream is

$$\gamma_i = \frac{\mathcal{E}_x}{\sigma^2 \|\mathbf{w}_{r,i}\|^2}, \quad i = 1, \ldots, N_t,$$

where $\mathbf{w}_{r,i}$ is the *i*th row of the zero-forcing or MMSE receiver **G**.

V-BLAST…

- Since this SNR is held hostage by the lower channel eigenvalues, ***the essence of V-BLAST is to combine a linear receiver with ordered successive interference cancellation.***
- Instead of detecting all Nt streams in parallel, they are detected iteratively.
- First, the ***strongest symbol stream is detected*** (using a ZF or MMSE receiver, as before).
- After these symbols are detected, they can be ***subtracted out from the composite received signal.***
- Then, the second strongest signal is detected, which now sees effectively Nt− 2 interfering streams.

V-BLAST…

- In general, the i^{th} detected stream experiences interference from only Nt − i of the transmit antennas
  - ➢ so that by the time the weakest symbol stream is detected, the vast majority of spatial interference has been removed.
- Employing the ordered successive interference cancellation lowers the block error rate by about a factor of 10 relative to a purely linear receiver
- or equivalently, decreases the required SNR by about 4dB.
- Despite its apparent simplicity, V-BLAST prototypes have shown spectral efficiencies above 20 bps/Hz

CLOSED-LOOP MIMO

- The potential gain from transmitter channel knowledge is quite significant in spatial multiplexing systems.
- First we will consider a simple theoretical example using ***singular value decomposition (SVD)*** that shows the potential gain of closed-loop spatial multiplexing methods.

SVD PRECODING AND POSTCODING

- A relatively straightforward way to see the gain of transmitter channel knowledge is by considering the *singular value decomposition* (SVD, or generalized eigenvalue decomposition) of the channel matrix **H**, which can be written as:

$$\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^*$$

  - where **U** and **V** are *unitary* and ∑ is a diagonal matrix of singular values.

SVD PRECODING AND POSTCODING

- As shown in fig, with linear operations at the transmitter and receiver, i.e., multiplying by **V** and **U***, respectively, the channel can be diagonalized.
- Mathematically, this can be confirmed by considering a decision vector **d** that should be close to the input symbol vector **b**.

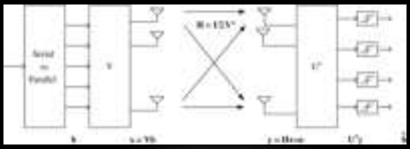


Figure : A MIMO system that has been diagonalized through SVD precoding

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SVD PRECODING AND POSTCODING…

- The decision vector can be written systematically as

$$\begin{aligned}\mathbf{d} &= \mathbf{U}^*\mathbf{y},\\ &= \mathbf{U}^*(\mathbf{Hx}+\mathbf{n}),\\ &= \mathbf{U}^*(\mathbf{U}\Sigma\mathbf{V}^*\mathbf{Vb}+\mathbf{n}),\\ &= \mathbf{U}^*\mathbf{U}\Sigma\mathbf{V}^*\mathbf{Vb}+\mathbf{U}^*\mathbf{n},\\ &= \Sigma\mathbf{b}+\mathbf{U}^*\mathbf{n}.\end{aligned}$$

  - which has diagonalized the channel and removed all the spatial interference without any matrix inversions or non-linear processing.
- Because U is unitary, U*n still has the same variance as n.
- Thus, the singular value approach does not result in noise enhancement, as did the open-loop linear techniques.
- Nevertheless, it shows the promise of closed-loop MIMO in achieving high performance at much lower complexity than the ML detector in open-loop MIMO.

LINEAR PRECODING AND POSTCODING

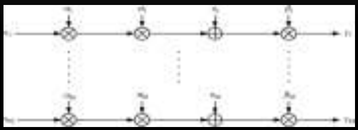
- The SVD was an illustrative example of how linear precoding and postcoding can diagonalize the MIMO channel matrix to provide up to min(N_r, N_t) dimensions to communicate data symbols through.
- More generally, the precoder and postcoder can be jointly designed based on a criteria such as the information capacity , the error probability, the detection MSE , or the received SNR.
- The general precoding formulation is

$$\mathbf{y} = \mathbf{G}(\mathbf{HFx}+\mathbf{n}),$$

  - where **x** and **y** are $M \times 1$, the postcoder matrix **G** is $M \times N_r$, the channel matrix **H** is $N_r \times N_t$, the precoder matrix **F** is $N_t \times M$, and **n** is $N_r \times 1$.
  - For the SVD example $M = \min(N_r, N_t)$, **G** = **U***, and **F** = **V**.

LINEAR PRECODING AND POSTCODING

- Regardless of the specific design criteria, the linear precoder and postcoder decompose the MIMO channel into a set of parallel subchannels as illustrated in figure below.



- Therefore, the received symbol for the *i*th subchannel can be expressed as:

$$y_i = \alpha_i\sigma_i\beta_i x_i + \beta_i n_i, \quad i = 1,\ldots,M,$$

  - where x_i and y_i are the transmitted and received symbols, respectively, with $E|x_i|^2 = \varepsilon_x$ as usual, σ_i are the singular values of **H**, α_i and β_i are the precoder and the postcoder weights, and n_i is the noise per subchannel

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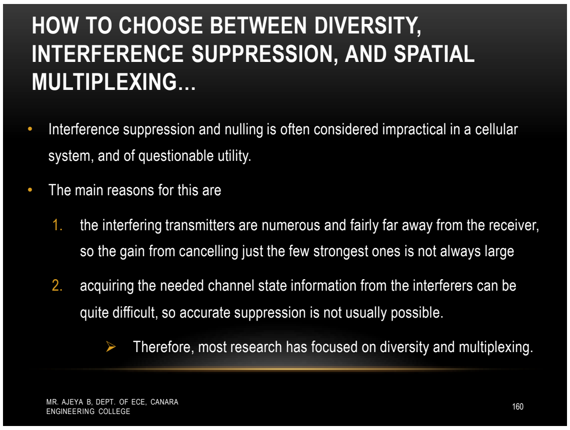
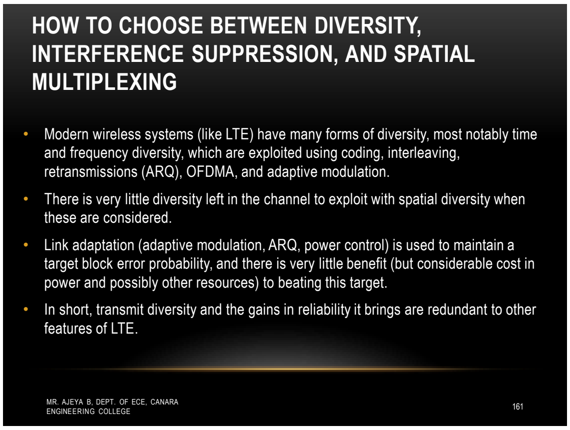
HOW TO CHOOSE BETWEEN DIVERSITY, INTERFERENCE SUPPRESSION, AND SPATIAL MULTIPLEXING

- Diversity provides robustness to fades and interference suppression provides robustness to interference.
- Neither increase the number of streams that can be sent, but they do increase the possible throughput on the stream that is sent by increasing the SINR = $S/(I + N)$.
- In particular, diversity increases and steadies S, while interference suppression (or nulling) reduces I.
- On the other hand, spatial multiplexing creates more parallel streams but does not necessarily increase the per-stream SINR.



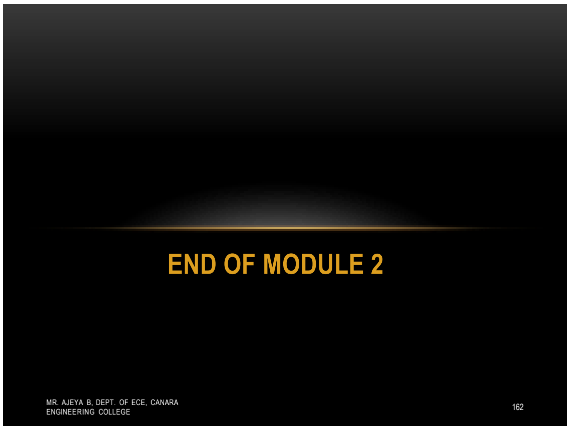
HOW TO CHOOSE BETWEEN DIVERSITY, INTERFERENCE SUPPRESSION, AND SPATIAL MULTIPLEXING…

- Interference suppression and nulling is often considered impractical in a cellular system, and of questionable utility.
- The main reasons for this are
  1. the interfering transmitters are numerous and fairly far away from the receiver, so the gain from cancelling just the few strongest ones is not always large
  2. acquiring the needed channel state information from the interferers can be quite difficult, so accurate suppression is not usually possible.
     - ➢ Therefore, most research has focused on diversity and multiplexing.



HOW TO CHOOSE BETWEEN DIVERSITY, INTERFERENCE SUPPRESSION, AND SPATIAL MULTIPLEXING

- Modern wireless systems (like LTE) have many forms of diversity, most notably time and frequency diversity, which are exploited using coding, interleaving, retransmissions (ARQ), OFDMA, and adaptive modulation.
- There is very little diversity left in the channel to exploit with spatial diversity when these are considered.
- Link adaptation (adaptive modulation, ARQ, power control) is used to maintain a target block error probability, and there is very little benefit (but considerable cost in power and possibly other resources) to beating this target.
- In short, transmit diversity and the gains in reliability it brings are redundant to other features of LTE.



END OF MODULE 2