Digital Image Processing
Module 5:
Image Segmentation

Element of Image Analysis

Preprocess
Image acquisition, restoration, and enhancement

⬇

Intermediate process
Image segmentation and feature extraction

⬇

High level process
Image interpretation and recognition

Importance of Image Segmentation

Image segmentation is used to separate an image into constituent parts based on some image attributes. Image segmentation is an important step in image analysis

Benefit

1. Image segmentation reduces huge amount of unnecessary data while retaining only importance data for image analysis
2. Image segmentation converts bitmap data into better structured data which is easier to be interpreted

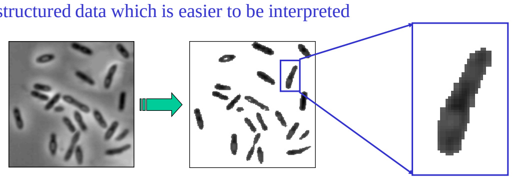
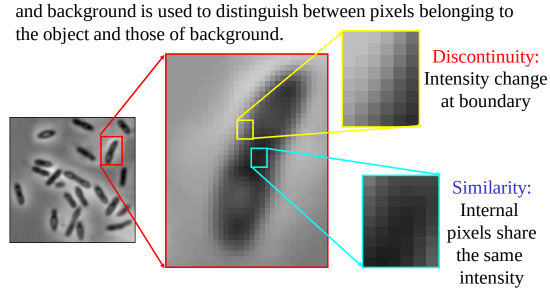


Image Attributes for Image Segmentation

1. Similarity properties of pixels inside the object are used to group pixels into the same set.

2. Discontinuity of pixel properties at the boundary between object and background is used to distinguish between pixels belonging to the object and those of background.



Spatial Filtering Application to Shape Detection

❖ One application of spatial filtering is shape detection: finding locations of objects with the desired shape.

❖ Unlike frequency selective masks that are designed based on the concept of frequency, shape detection masks are derived from the shapes to be detected themselves.

❖ A mask for shape detection usually contains the shape or a part of the shape to be detected.

❖ The location that is most correlated to the mask is the location where the highest filter response occurs. The shape is most likely to exist there.

Point Detection

❖ We can use Laplacian masks for point detection.

❖ Laplacian masks have the largest coefficient at the center of the mask while neighbor pixels have an opposite sign.

-1	-1	-1
-1	8	-1
-1	-1	-1

0	-1	0
-1	4	-1
0	-1	0

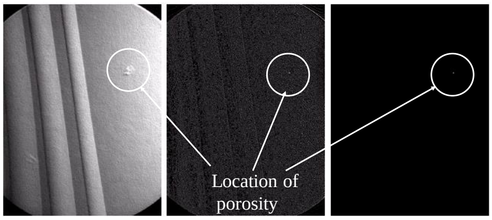
❖ This mask will give the high response to the object that has the similar shape as the mask such as isolated points.

❖ Notice that sum of all coefficients of the mask is equal to zero. This is due to the need that the response of the filter must be zero inside a constant intensity area

Point Detection

Point detection can be done by applying the thresholding function:

$$g(x,y)=\begin{cases}1 & |\nabla f(x,y)|\geq T\\ 0 & \text{otherwise}\end{cases}$$



X-ray image of the turbine blade with porosity | Laplacian image | After thresholding

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Line Detection

❖ Similar to point detection, line detection can be performed using the mask the has the shape look similar to a part of a line

❖ There are several directions that the line in a digital image can be.

❖ For a simple line detection, 4 directions that are mostly used are Horizontal, +45 degree, vertical and –45 degree.

Horizontal

–1	–1	–1
2	2	2
–1	–1	–1

+45°

–1	–1	2
–1	2	–1
2	–1	–1

Vertical

–1	2	–1
–1	2	–1
–1	2	–1

–45°

2	–1	–1
–1	2	–1
–1	–1	2

Line detection masks

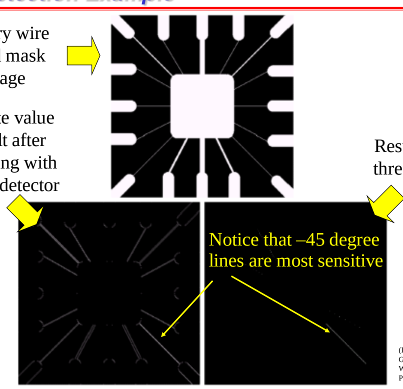
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Line Detection Example

Binary wire bond mask image

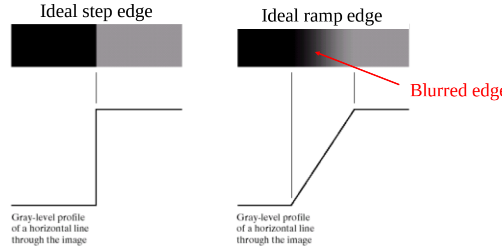
Absolute value of result after processing with -45 line detector

Result after thresholding



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

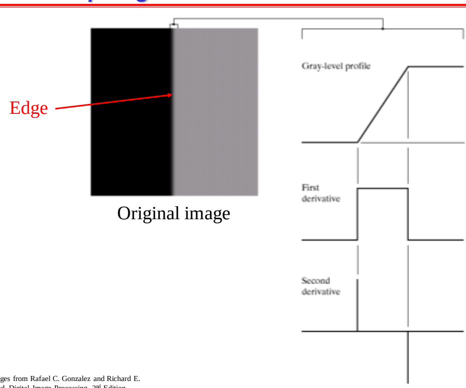
Edges



Generally, objects and background have different intensities. Therefore, Edges of the objects are the areas where abrupt intensity changes occur.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Ideal Ramp Edges and its Derivatives



Original image

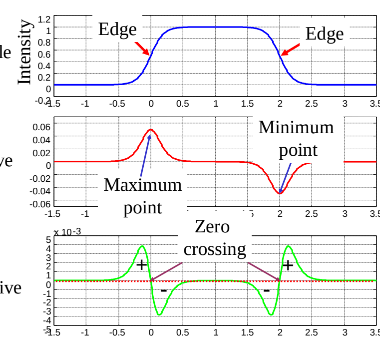
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Smoothed Step Edge and Its Derivatives

Gray level profile

The 1st derivative

The 2nd derivative



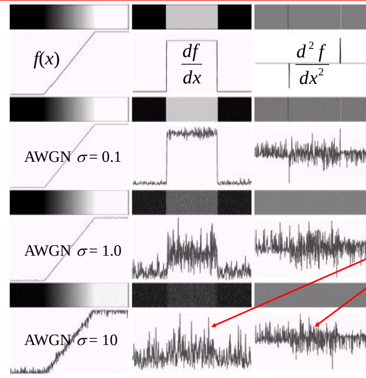
Derivative Based Edge Detection

❖ From the previous slide, we can conclude that:
Local maxima of the absolute of the 1st derivative and Zero crossing of the 2nd derivative occur at edges.

❖ Therefore, for detecting edges, we can apply zero crossing detection to the 2nd derivative image or thresholding the absolute of the 1st derivative image.

❖ Nevertheless, derivative operator is very sensitive to noise as we will see in the next slide.

Noisy Edges and Derivatives



Derivative operator is a highpass filter and thus enhances noise.

Edge responses are buried by noise.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Masks for Estimating Partial Derivatives

Normally, the mask for estimating partial derivative is anti-symmetry with respect to the orthogonal axis

Roberts

−1	0		0	−1
0	1		1	0

Prewitt

−1	−1	−1		−1	0	1
0	0	0		−1	0	1
1	1	1		−1	0	1

Sobel

−1	−2	−1		−1	0	1
0	0	0		−2	0	2
1	2	1		−1	0	1

For example, the Sobel mask for computing $\frac{\partial f}{\partial x}$ is anti-symmetry with respect to the y-axis. It has the positive sign on the right side and negative sign on the left side.

Notice that sum of all coefficients is equal to zero to make sure that the response of a constant intensity area is zero.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Masks for Detecting Diagonal Edges

Prewitt

0	1	1		−1	−1	0
−1	0	1		−1	0	1
−1	−1	0		0	1	1

Sobel

0	1	2		−2	−1	0
−1	0	1		−1	0	1
−2	−1	0		0	1	2

The mask for detecting -45-degree edges is anti-symmetry with respect to the –45-degree lines while the mask for detecting 45-degree edges is anti-symmetry with respect to the 45-degree lines.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Detection of Discontinuities Gradient Operators

An **image gradient** is a directional change in the intensity or color in an **image**.

- **First-order derivatives:**
  - The gradient of an image $f(x,y)$ at location (x,y) is defined as the vector:

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

  - The magnitude of this vector: $\nabla f = \mathrm{mag}(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2\right]^{1/2}$
  - The direction of this vector: $\alpha(x,y) = \tan^{-1}\left(\frac{G_x}{G_y}\right)$

Detection of Discontinuities Gradient Operators

Roberts cross-gradient operators ➡

−1	0		0	−1
0	1		1	0

Roberts

Prewitt operators ➡

−1	−1	−1		−1	0	1
0	0	0		−1	0	1
1	1	1		−1	0	1

Prewitt

Sobel operators ➡

−1	−2	−1		−1	0	1
0	0	0		−2	0	2
1	2	1		−1	0	1

Sobel

Detection of Discontinuities
Gradient Operators

Prewitt masks for detecting diagonal edges

0	1	1	−1	−1	0
−1	0	1	−1	0	1
−1	−1	0	0	1	1

Prewitt

Sobel masks for detecting diagonal edges

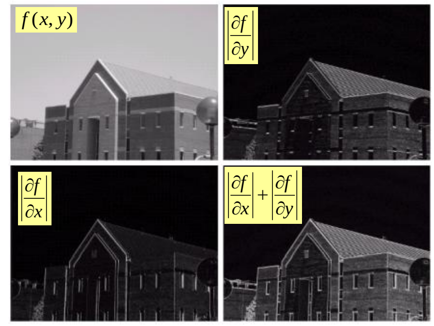
0	1	2	−2	−1	0
−1	0	1	−1	0	1
−2	−1	0	0	1	2

Sobel

a b
c d

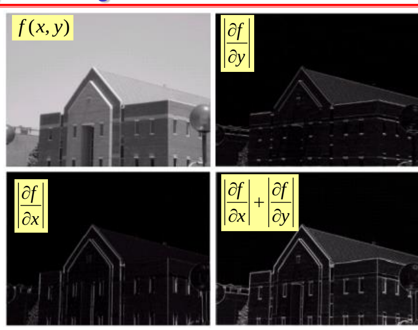
FIGURE 10.9 Prewitt and Sobel masks for detecting diagonal edges.

Example of Image Gradient



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

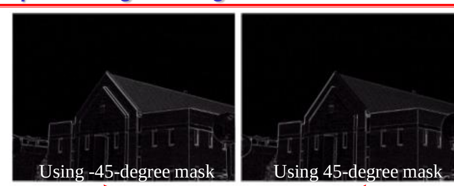
Example of Image Gradient



Note: the original image is smoothed by a 5x5 moving average mask first.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example of Diagonal Edges



0	1	2	−2	−1	0
−1	0	1	−1	0	1
−2	−1	0	0	1	2

Sobel

Note: the original image is smoothed by a 5x5 moving average mask first.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Detection of Discontinuities
Gradient Operators

- Second-order derivatives: (The Laplacian)
  - The Laplacian of an 2D function $f(x,y)$ is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

  - Two forms in practice:

FIGURE 10.13 Laplacian masks used to implement Eqs. (10.1-14) and (10.1-15), respectively.

0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

Laplacian Masks

The Laplacian masks are used to estimate the Laplacian image:

$$\nabla^2 P = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}$$

Ideally, the Laplacian mask must be directional invariant: symmetry in all direction (radially symmetry). However, for 3x3 masks, there are Only 8 possible directions. Hence, we can use the following masks:

0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Detection of Discontinuities Gradient Operators

- Consider the function:

A Gaussian function

$$h(r) = -e^{-\frac{r^2}{2\sigma^2}} \quad \text{where} \quad r^2 = x^2 + y^2$$

and σ : the standard deviation

- The Laplacian of h is

$$\nabla^2 h(r) = -\left[\frac{r^2 - \sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$

The Laplacian of a Gaussian (LoG)

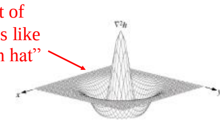
- The Laplacian of a Gaussian sometimes is called the Mexican hat function. It also can be computed by smoothing the image with the Gaussian smoothing mask, followed by application of the Laplacian mask.

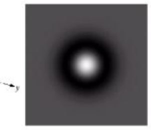
Laplacian Masks

For a large scale Laplacian mask, we can use a Laplacian of Gaussian (LOG) as a mask:

$$\nabla^2 G(x,y) = -\left[\frac{x^2 + y^2 - \sigma^2}{\sigma^4}\right] e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}$$

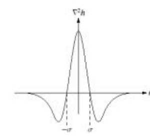
Surface plot of LOG, Looks like a "Mexican hat"





LOG image

Cross section of LOG



0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

5x5 LOG mask

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example of Laplacian Image

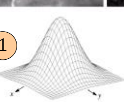
The angiogram image (blood vessels)

$|\nabla P|$

We can compute the Laplacian image by:

1. Smooth the image by the Gaussian mask
2. Compute the Laplacian image using the mask

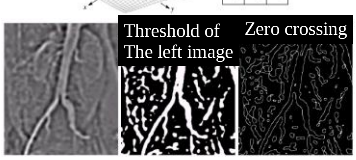
1



-1	-1	-1
-1	8	-1
-1	-1	-1

2

$\nabla^2 G * P$

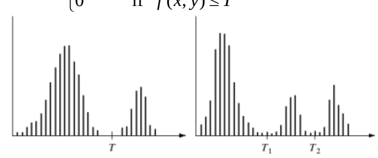


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Thresholding

- Assumption: the range of intensity levels covered by objects of interest is different from the background.

$$g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > T \\ 0 & \text{if } f(x,y) \le T \end{cases}$$

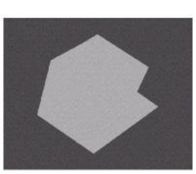


a b

Single threshold Multiple threshold

FIGURE 10.26 (a) Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.

Thresholding The Role of Illumination



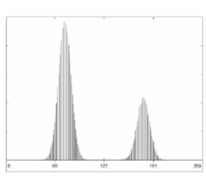


FIGURE 10.27 (a) Computer generated reflectance function. (b) Histogram of reflectance function.

Nonuniform Illumination Problem

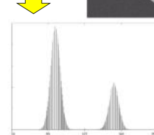
Reflectance Function $r(x,y)$

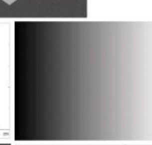
An image can be expressed as

$$f(x,y) = i(x,y)r(x,y)$$

$i(x,y)$ = illumination component
$r(x,y)$ = reflectance component

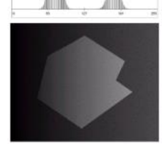
Histogram





Illumination Function $i(x,y)$

$f(x,y)$



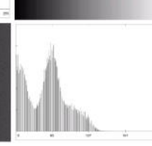
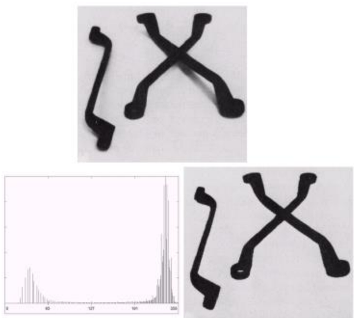


Image histogram

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

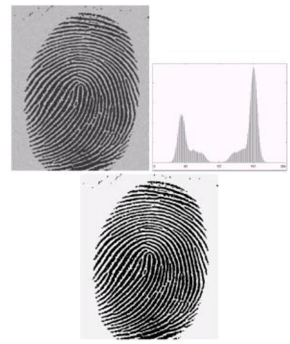
Thresholding
Basic Global Thresholding



a
b c

FIGURE 10.28 (a) Original image. (b) Image histogram. (c) Result of global thresholding with T midway between the maximum and minimum gray levels.

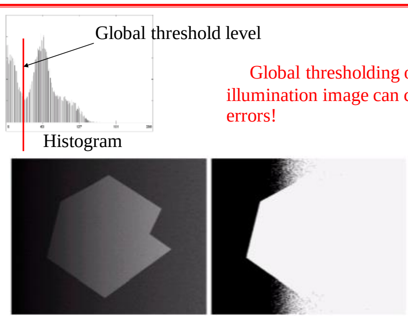
Thresholding
Basic Global Thresholding



a b
c

FIGURE 10.29 (a) Original image. (b) Image histogram. (c) Result of segmentation with the threshold estimated by iteration. (Original courtesy of the National Institute of Standards and Technology.)

Nonuniform Illumination and Global Thresholding



Global thresholding of nonuniform illumination image can cause huge errors!

Nonuniform illumination image

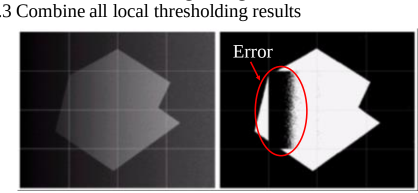
Global thresholding result

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Nonuniform Illumination and Local Thresholding

Local thresholding:

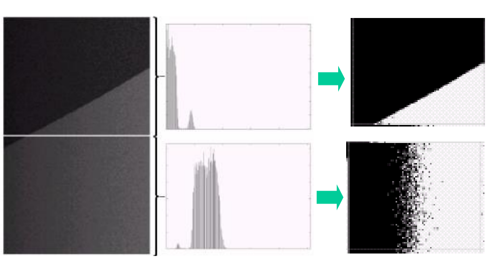
1. Divide an image into subimages.
2. Threshold each subimage independently
   - 2.1 Compute histogram of each subimage and select a suitable threshold value for each subimage
   - 2.2 threshold each subimage using a threshold value in 2.1
   - 2.3 Combine all local thresholding results



16 subimages

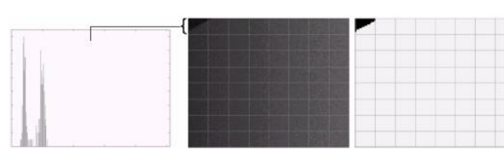
Result of local thresholding

Thresholding
Basic Adaptive Thresholding



How to solve this problem?

Thresholding
Basic Adaptive Thresholding



Answer: subdivision

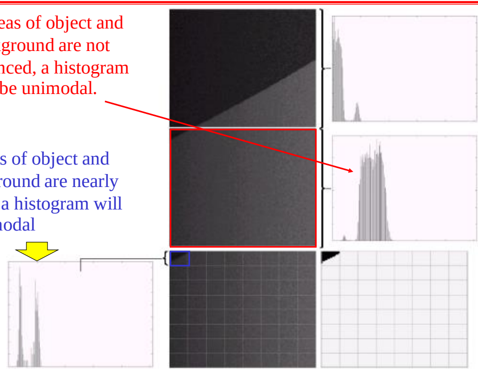
a b
c
c d f

FIGURE 10.31 (a) Properly and improperly segmented subimages from Fig. 10.30. (b)–(c) Corresponding histograms. (d) Further subdivision of the improperly segmented subimage. (e) Histogram of small subimage at top, left. (f) Result of adaptively segmenting (d).

Histogram of Subimages and Local Thresholding

If areas of object and background are not balanced, a histogram will be unimodal.

If areas of object and background are nearly equal, a histogram will be bimodal

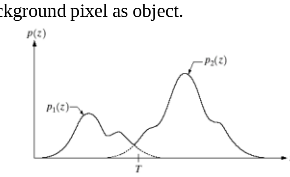


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.)

Thresholding
Optimal Global and Adaptive Thresholding

- This method treats pixel values as probability density functions.
- The goal of this method is to minimize the probability of misclassifying pixels as either object or background.
- There are two kinds of error:
  - mislabeling an object pixel as background, and
  - mislabeling a background pixel as object.

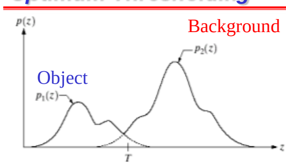
FIGURE 10.32 Gray-level probability density functions of two regions in an image.



Optimum Thresholding

Background

Object



$p_1(z)$ = PDF of object pixels
$p_2(z)$ = PDF of background pixels

Error due to background pixels classified as object pixels is :
$$E_1(T) = \int_{-\infty}^{T} p_2(z)dz$$

Error due to object pixels classified as background pixels is:
$$E_2(T) = \int_{T}^{\infty} p_1(z)dz$$

Total error = $E(T) = P_2E_1(T) + P_1E_2(T)$

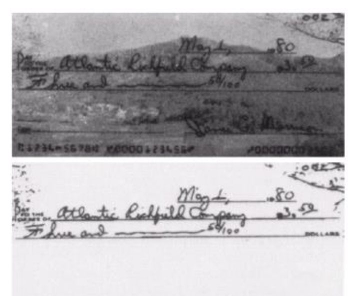
P_1 = Probability of occurrence of object pixels
P_2 = Probability of occurrence of background pixels

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.)

Thresholding
Use of Boundary Characteristics

a
b

FIGURE 10.37 (a) Original image. (b) Image segmented by local thresholding. (Courtesy of IBM Corporation.)



Thresholding
Thresholds Based on Several Variables

Color image

a b c

FIGURE 10.39 (a) Original color image shown as a monochrome picture. (b) Segmentation of pixels with colors close to facial tones. (c) Segmentation of red components.

Region-Based Segmentation

- Edges and thresholds sometimes do not give good results for segmentation.
- Region-based segmentation is based on the connectivity of similar pixels in a region.
  - Each region must be uniform.
  - Connectivity of the pixels within the region is very important.
- There are two main approaches to region-based segmentation: region growing and region splitting.

Basic Formulation

Let R represent the entire image region. Segmentation partitions R into n subregions, $R_1, R_2, \ldots, R_n$, such that:

a) $\bigcup_{i=1}^{n} R_i = R$

b) R_i is a connected region, $i = 1, 2, \ldots, n$.

c) $R_i \cap R_j = \phi$ for all i and $j, i \neq j$

d) $P(R_i) = TRUE$ for $i = 1, 2, \ldots, n$.

e) $P(R_i \cup R_j) = FALSE$ for $i \neq j$.

a) Every pixel must be in a region

b) Points in a region must be connected.

c) Regions must be disjoint.

d) All pixels in a region satisfy specific properties.

e) Different regions have different properties.

How do we form regions?

- Region Growing
- Region Merging
- Region Splitting
- Split and Merge
- Watershed
- ...

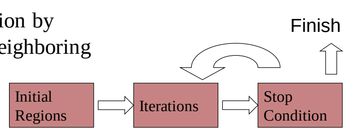
0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

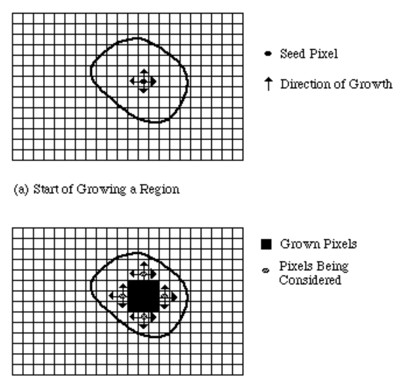
What a computer sees

Region growing

- Groups pixels into larger regions.
- Starts with a **seed** region.
- **Grows** region by **merging** neighboring pixels.

- Iterative process
  - How to start?
  - How to iterate?
  - When to stop?





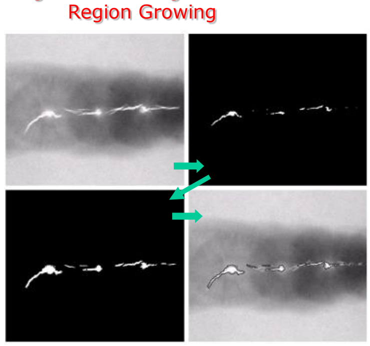
(a) Start of Growing a Region

(b) Growing Process After a Few Iterations

Region-Based Segmentation Region Growing

a b
c d

FIGURE 10.40 (a) Image showing defective welds. (b) Seed points. (c) Result of region growing. (d) Boundaries of segmented defective welds (in black). (Original image courtesy of X-TEK Systems, Ltd.).



Region-Based Segmentation Region Growing

- Fig. 10.41 shows the histogram of Fig. 10.40 (a). It is difficult to segment the defects by thresholding methods. (Applying region growing methods are better in this case.)

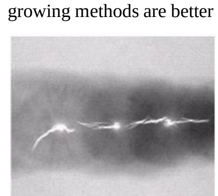


Figure 10.40(a)

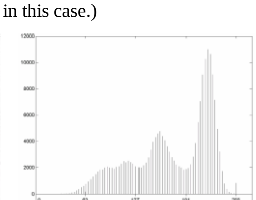


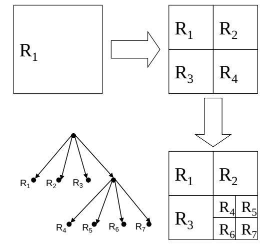
Figure 10.41

Region-Based Segmentation
Region Splitting and Merging

- Region splitting is the opposite of region growing.
  - First there is a large region (possible the entire image).
  - Then a predicate (measurement) is used to determine if the region is uniform.
  - If not, then the method requires that the region be split into two regions.
  - Then each of these two regions is independently tested by the predicate (measurement).
  - This procedure continues until all resulting regions are uniform.

Region splitting

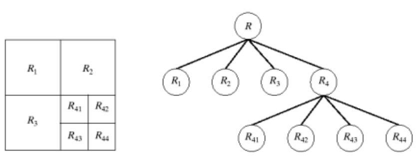
- Algorithm
  - One initial set that includes the **whole image**.
  - **Similarity criteria**.
  - Iteratively **split** regions into sub-regions.
  - Stop when no more splittings are possible.



Region-Based Segmentation
Region Splitting

- The main problem with region splitting is determining where to split a region.
- One method to divide a region is to use a quadtree structure.
- Quadtree: a tree in which nodes have exactly four descendants.

a b
FIGURE 10.42
(a) Partitioned image.
(b) Corresponding quadtree.



Region merging

- Algorithm
  - Divide image into an initial set of regions.
    - One region per pixel.
  - Define a **similarity criteria** for merging regions.
  - **Merge** similar regions.
  - Repeat previous step until no more merge operations are possible.

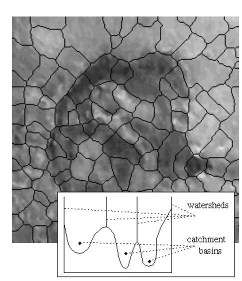
Region-Based Segmentation
Region Splitting and Merging

- The split and merge procedure:
  - Split into four disjoint quadrants any region R_i for which $P(R_i)$ = FALSE.
  - Merge any adjacent regions R_j and R_k for which $P(R_j UR_k)$ = TRUE. (the quadtree structure may not be preserved)
  - Stop when no further merging or splitting is possible.

a b c
FIGURE 10.43
(a) Original image. (b) Result of split and merge procedure.
(c) Result of thresholding (a).

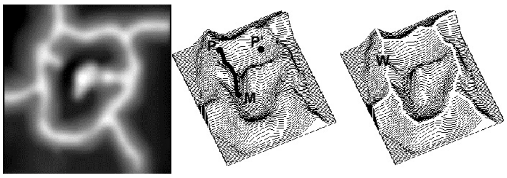
The *Watershed* Transform

- Geographical inspiration.
  - Shed water over rugged terrain.
  - Each lake corresponds to a region.
- Characteristics
  - Computationally complex.
  - Great flexibility in segmentation.
  - Risk of over-segmentation.



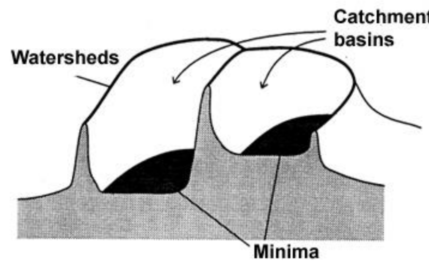
The Drainage Analogy

- Two points are in the same region if they drain to the same point.



Courtesy of Dr. Peter Yim at National Institutes of Health, Bethesda, MD

The Immersion Analogy



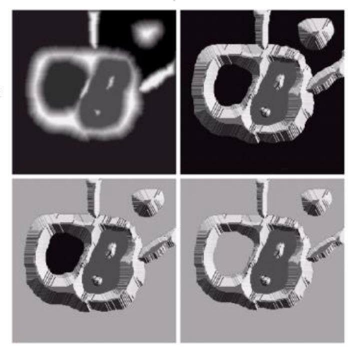
Segmentation by Morphological Watersheds

- The concept of watersheds is based on visualizing an image in three dimensions: two spatial coordinates versus gray levels.
- In such a topographic interpretation, we consider three types of points:
  - (a) points belonging to a regional minimum
  - (b) points at which a drop of water would fall with certainty to a single minimum
  - (c) points at which water would be equally likely to fall to more than one such minimum
- The principal objective of segmentation algorithms based on these concepts is to find the watershed lines.

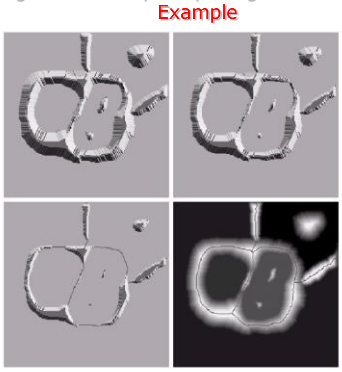
Segmentation by Morphological Watersheds Example

a b
c d

FIGURE 10.44 (a) Original image. (b) Topographic view. (c)–(d) Two stages of flooding.



Segmentation by Morphological Watersheds Example



e f
g h

FIGURE 10.44 *(Continued)* (e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

Segmentation by Morphological Watersheds Example

a b
c d

FIGURE 10.46 (a) Image of blobs. (b) Image gradient. (c) Watershed lines. (d) Watershed lines superimposed on original image. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)