Digital Image Processing
Module 3
Image Restoration

1

Preview

- Goal of image restoration
  - Improve an image in some predefined sense
  - Difference with image enhancement ?
- Features
  - Image restoration v.s image enhancement
  - Objective process v.s. subjective process
  - A prior knowledge v.s heuristic process
  - A prior knowledge of the degradation phenomenon is considered
  - Modeling the degradation and apply the inverse process to recover the original image

2

Preview (cont.)

- Target
  - Degraded digital image
  - Sensor, digitizer, display degradations are less considered
- Spatial domain approach
- Frequency domain approach

3

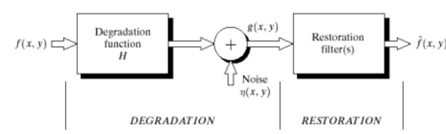
Unit Outline

- A model of the image degradation / restoration process
- Noise models
- Restoration in the presence of noise only – spatial filtering
- Periodic noise reduction by frequency domain filtering
- Linear, position-invariant degradations
- Estimating the degradation function
- Inverse filtering

4

Concept of Image Restoration

Image restoration is to restore a degraded image back to the original image while image enhancement is to manipulate the image so that it is suitable for a specific application.



Degradation model:

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

where $h(x,y)$ is a system that causes image distortion and $\eta(x,y)$ is noise.

5

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Noise models

- Source of noise
  - Image acquisition (digitization)
  - Image transmission
- Spatial properties of noise
  - Statistical behavior of the gray-level values of pixels
  - Noise parameters, correlation with the image
- Frequency properties of noise
  - Fourier spectrum
  - Ex. white noise (a constant Fourier spectrum)

6

Noise Models

Noise cannot be predicted but can be approximately described in statistical way using the probability density function (PDF)

Gaussian noise:

$$p(z) = \frac{1}{2\pi\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

Rayleigh noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for z < a} \end{cases}$$

Erlang (Gamma) noise

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!}(z-a)e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

7

Noise Models (cont.)

Exponential noise

$$p(z) = ae^{-az}$$

Uniform noise

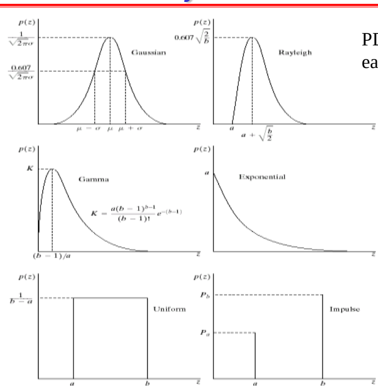
$$p(z) = \begin{cases} \frac{1}{\text{b - a}} & \text{for } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

Impulse (salt & pepper) noise

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

8

PDF: Statistical Way to Describe Noise



PDF tells how much each z value occurs.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.)

9

Noise probability density functions

- Noises are taken as random variables
- Random variables
  - Probability density function (PDF)

10

Gaussian noise

- Math. tractability in spatial and frequency domain
- Electronic circuit noise and sensor noise

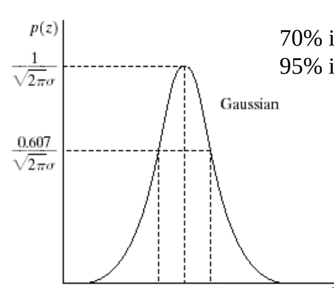
$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

mean   variance

Note: $\int_{-\infty}^{\infty} p(z)dz = 1$

11

Gaussian noise (PDF)



70% in [(μ–σ), (μ+σ)]
95% in [(μ–2σ), (μ+2σ)]

12

Uniform noise

- Less practical, used for random number generator

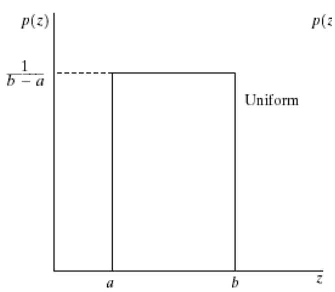
$$p(z)=\begin{cases}\frac{1}{b-a} & \text{if } a\le z\le b\\ 0 & \text{otherwise}\end{cases}$$

Mean: $\mu=\frac{a+b}{2}$

Variance: $\sigma^2=\frac{(b-a)^2}{12}$

13

Uniform Noise PDF



14

Impulse (salt-and-pepper) nosie

- Quick transients, such as faulty switching during imaging

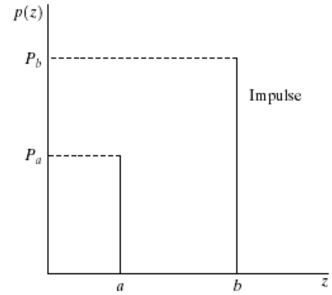
$$p(z)=\begin{cases}P_a & \text{for } z=a\\ P_b & \text{for } z=b\\ 0 & \text{otherwise}\end{cases}$$

If either P_a or P_b is zero, it is called *unipolar*. Otherwise, it is called bipoloar.

•In practical, impulses are usually stronger than image signals. Ex., a=0(black) and b=255(white) in 8-bit image.

15

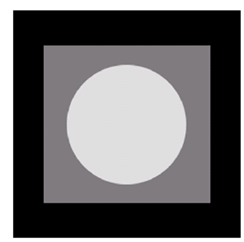
Impulse (salt-and-pepper) nosie PDF



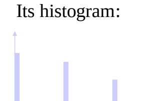
16

Test for noise behavior

- Test pattern



Its histogram:



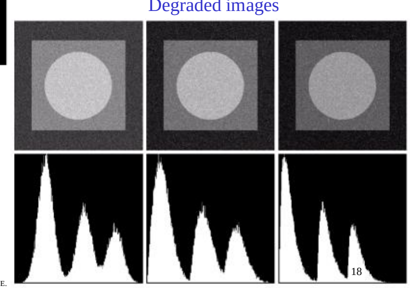
17

Image Degradation with Additive Noise

$g(x,y)=f(x,y)+\eta(x,y)$

Original image

Degraded images



Histogram

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

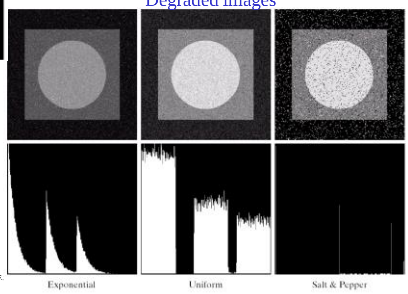
18

Image Degradation with Additive Noise (cont.)

$$g(x, y) = f(x, y) + \eta(x, y)$$

Original image

Degraded images



Histogram

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Estimation of noise parameters

- Periodic noise
  - Observe the frequency spectrum
- Random noise with unknown PDFs
  - Case 1: imaging system is available
    - Capture images of "flat" environment
  - Case 2: noisy images available
    - Take a strip from constant area
    - Draw the histogram and observe it
    - Measure the mean and variance

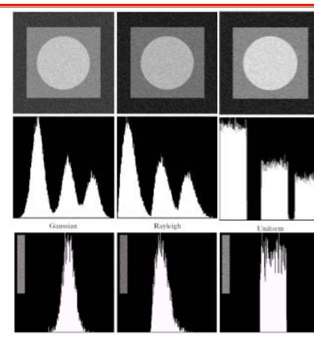
22

Periodic noise

- Source: electrical or electromechanical interference during image acquisition
- Characteristics
  - Spatially dependent
  - Periodic – easy to observe in frequency domain
- Processing method
  - Suppressing noise component in frequency domain

20

Estimation of Noise



We cannot use the image histogram to estimate noise PDF.

It is better to use the histogram of one area of an image that has constant intensity to estimate noise PDF.

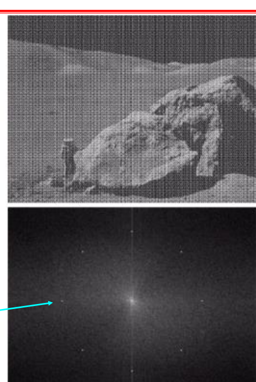
23

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Periodic Noise

a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)



Periodic noise looks like dots In the frequency domain

21

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Restoration in the presence of Noise Only- Spatial Filtering

When only degradation present in an image is noise,

Degradation model:

$$g(x, y) = f(x, y) * h(x, y) + \eta(x, y)$$

To remove this part

Spatial filtering is the method of choice in situations when only additive noise is present.
Enhancement and Restoration become almost indistinguishable disciplines in this particular case.

24

Mean Filters (Noise Reduction Spatial Filters)

Arithmetic mean filter or moving average filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Geometric mean filter

$$\hat{f}(x,y) = \left(\prod_{(s,t)\in S_{xy}} g(s,t) \right)^{\frac{1}{mn}}$$

mn = size of moving window

Achieves smoothing comparable to arithmetic mean filter, but it tends to lose less image detail in the process.

25

Geometric Mean Filter: Example

Original image

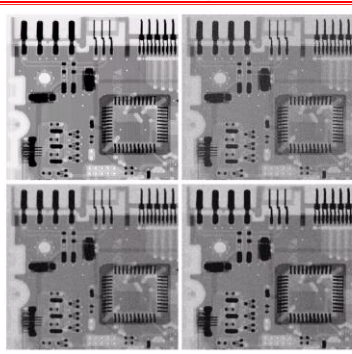


Image corrupted by AWGN

Image obtained using a 3x3 arithmetic mean filter

Image obtained using a 3x3 geometric mean filter

AWGN: Additive White Gaussian Noise

26

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Harmonic and Contraharmonic Filters

Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise but fails for pepper noise
Works well for Gaussian Noise

mn = size of moving window

Contraharmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Reduces salt and pepper noise,
Positive Q is suitable for eliminating pepper noise.
Negative Q is suitable for eliminating salt noise.
Cannot do both simultaneously

Q = the filter order

For Q = 0, the filter reduces to an arithmetic mean filter.
For Q = -1, the filter reduces to a harmonic mean filter.

27

Contraharmonic Filters: Example

Image corrupted by pepper noise with prob. = 0.1

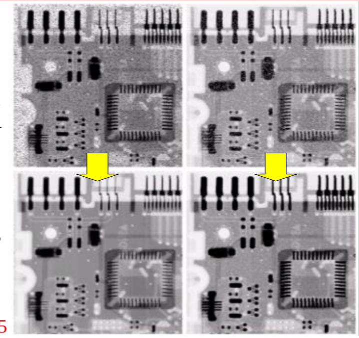


Image corrupted by salt noise with prob. = 0.1

Image obtained using a 3x3 contra-harmonic mean filter With Q = 1.5

Image obtained using a 3x3 contra-harmonic mean filter With Q=-1.5

28

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Contraharmonic Filters: Incorrect Use Example

Image corrupted by pepper noise with prob. = 0.1

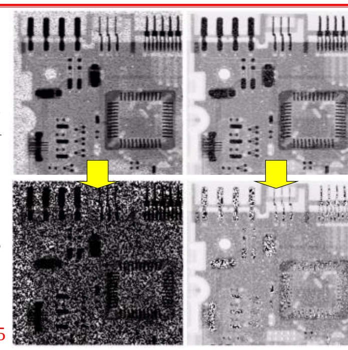


Image corrupted by salt noise with prob. = 0.1

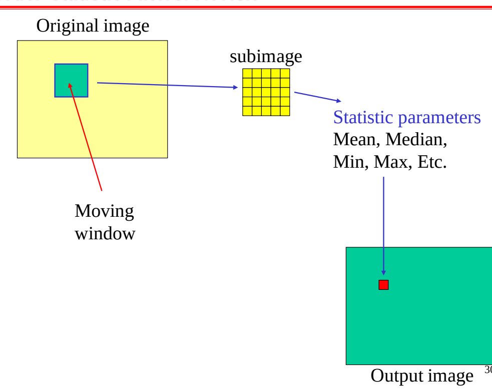
Image obtained using a 3x3 contra-harmonic mean filter With Q=-1.5

Image obtained using a 3x3 contra-harmonic mean filter With Q=1.5

29

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Order-Statistic Filters: Revisit



30

Order-Statistics Filters

Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}} \{g(s,t)\}$$

Max filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\max} \{g(s,t)\}$$

→ Reduce "dark" noise (pepper noise)

Min filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\min} \{g(s,t)\}$$

→ Reduce "bright" noise (salt noise)

Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left(\underset{(s,t)\in S_{xy}}{\max} \{g(s,t)\} + \underset{(s,t)\in S_{xy}}{\min} \{g(s,t)\} \right)$$

31

Max and Min Filters: Example

Image corrupted by pepper noise with prob. = 0.1

Image corrupted by salt noise with prob. = 0.1

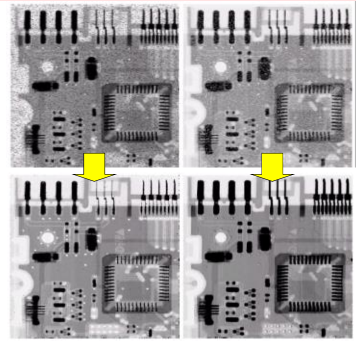


Image obtained using a 3x3 max filter

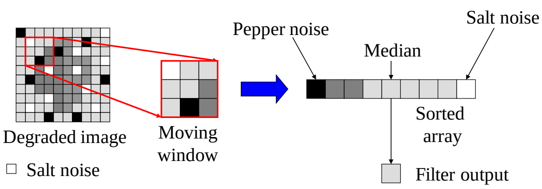
Image obtained using a 3x3 min filter

34

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Median Filter : How it works

A median filter is good for removing impulse, isolated noise



☐ Salt noise

■ Pepper noise

Normally, impulse noise has high magnitude and is isolated. When we sort pixels in the moving window, noise pixels are usually at the ends of the array.

Therefore, it's rare that the noise pixel will be a median value.

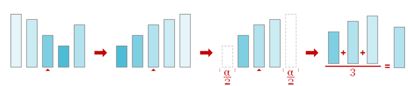
32

Alpha-trimmed Mean Filter

Formula:

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

where $g_r(s,t)$ represent the remaining mn-d pixels after removing the $d/2$ highest and $d/2$ lowest values of $g(s,t)$.

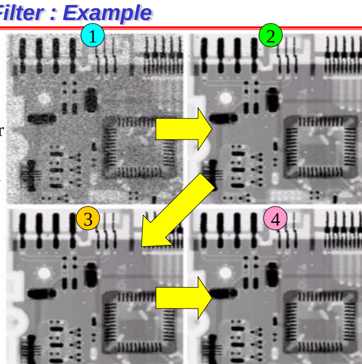


This filter is useful in situations involving multiple types of noise such as a combination of salt-and-pepper and Gaussian noise.

35

Median Filter : Example

Image corrupted by salt-and-pepper noise with p_a=p_b= 0.1



Images obtained using a 3x3 median filter

33

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Alpha-trimmed Mean Filter: Example

Image corrupted by additive uniform noise

Image additionally corrupted by additive salt-and-pepper noise

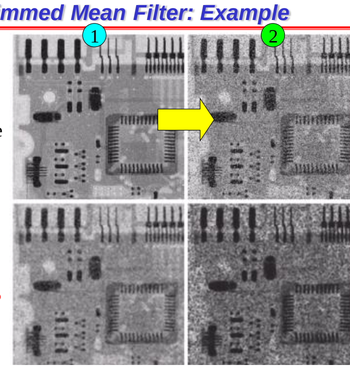


Image 2 obtained using a 5x5 arithmetic mean filter

Image 2 obtained using a 5x5 geometric mean filter

36

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Alpha-trimmed Mean Filter: Example (cont.)

Image corrupted by additive uniform noise

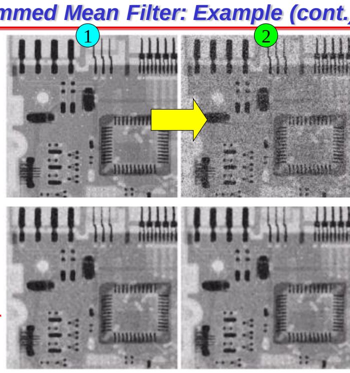


Image additionally corrupted by additive salt-and-pepper noise

Image 2 obtained using a 5x5 median filter

Image 2 obtained using a 5x5 alpha-trimmed mean filter with $d = 5$

37

Alpha-trimmed Mean Filter: Example (cont.)

Image obtained using a 5x5 arithmetic mean filter

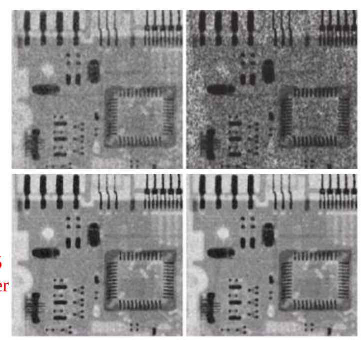


Image obtained using a 5x5 geometric mean filter

Image obtained using a 5x5 median filter

Image obtained using a 5x5 alpha-trimmed mean filter with $d = 5$

38

Adaptive Filters

- The filters discussed so far are applied to an entire image without any regard for how image characteristics vary from one point to another
- The behaviour of **adaptive filters** changes depending on the characteristics of the image inside the filter region

39

Adaptive Filter

General concept:

-Filter behavior depends on statistical characteristics of local areas inside *mxn* moving window

- More complex but superior performance compared with "fixed" filters

Statistical characteristics:

Local mean:

$$m_L = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Noise variance:

$$\sigma_\eta^2$$

Local variance:

$$\sigma_L^2 = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} (g(s,t) - m_L)^2$$

40

Adaptive, Local Noise Reduction Filter

Purpose: want to preserve edges

Concept:

1. If σ_η^2 is zero, → No noise
   the filter should return $g(x,y)$ because $g(x,y) = f(x,y)$
2. If σ_L^2 is high relative to σ_η^2, → Edges (should be preserved),
   the filter should return the value close to $g(x,y)$
3. If $\sigma_L^2 = \sigma_\eta^2$, → Areas inside objects
   the filter should return the arithmetic mean value m_L

Formula:

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left(g(x,y) - m_L\right)$$

41

Adaptive Noise Reduction Filter: Example

Image corrupted by additive Gaussian noise with zero mean and σ^2=1000

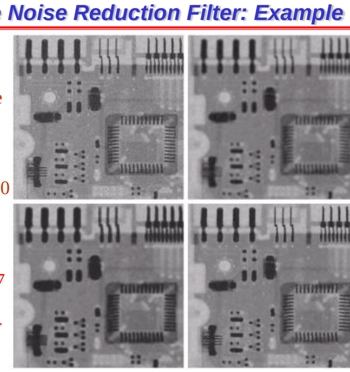


Image obtained using a 7x7 arithmetic mean filter

Image obtained using a 7x7 geometric mean filter

Image obtained using a 7x7 adaptive noise reduction filter

42

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.)

Adaptive Median Filtering

- The median filter performs relatively well on impulse noise as long as the spatial density of the impulse noise is not large
- The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise
- The key insight in the adaptive median filter is that the filter size changes depending on the characteristics of the image

43

Adaptive Median Filtering (cont…)

The key to understanding the algorithm is to remember that the adaptive median filter has three purposes:

- Remove impulse noise
- Provide smoothing of other noise
- Reduce distortion

44

Adaptive Median Filter

Purpose: want to remove impulse noise while preserving edges

Algorithm: Level *A*:
$A1 = z_{median} - z_{min}$
$A2 = z_{median} - z_{max}$
If A1 > 0 and A2 < 0, goto level B
Else increase window size
If window size <= S_{max} repeat level A
Else return z_{xy}

Level *B*:
$B1 = z_{xy} - z_{min}$
$B2 = z_{xy} - z_{max}$
If B1 > 0 and B2 < 0, return z_{xy}
Else return z_{median}

where
z_{min} = minimum gray level value in S_{xy}
z_{max} = maximum gray level value in S_{xy}
z_{median} = median of gray levels in S_{xy}
z_{xy} = gray level value at pixel (x,y)
S_{max} = maximum allowed size of S_{xy}

45

Adaptive Median Filter: How it works

Level *A*: $A1 = z_{median} - z_{min}$
$A2 = z_{median} - z_{max}$
If A1 > 0 and A2 < 0, goto level B
Else → Window is not big enough
increase window size
If window size <= S_{max} repeat level A
Else return z_{xy}

} Determine whether z_{median} is an impulse or not

Level *B*: → z_{median} is not an impulse
$B1 = z_{xy} - z_{min}$
$B2 = z_{xy} - z_{max}$
If B1 > 0 and B2 < 0,
return z_{xy} → to preserve original details
Else
return z_{median} → to remove impulse

} Determine whether z_{xy} is an impulse or not
→ z_{xy} is not an impulse

46

Adaptive Median Filter: Example

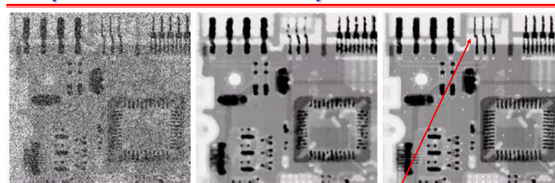


Image corrupted by salt-and-pepper noise with $p_a = p_b = 0.25$

Image obtained using a 7x7 median filter

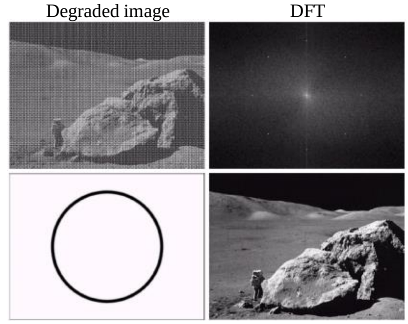
Image obtained using an adaptive median filter with $S_{max} = 7$

More small details are preserved

47

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Periodic Noise Reduction by Freq. Domain Filtering



Degraded image — DFT

Band reject filter — Restored image

Periodic noise can be reduced by setting frequency components corresponding to noise to zero.

48

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Band Reject Filters

Use to eliminate frequency components in some bands

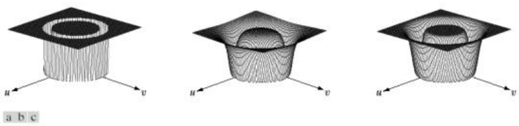
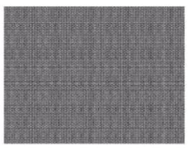


FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Periodic noise from the previous slide that is Filtered out.

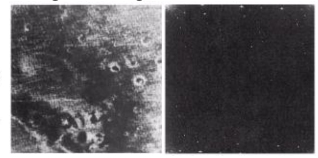
49

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

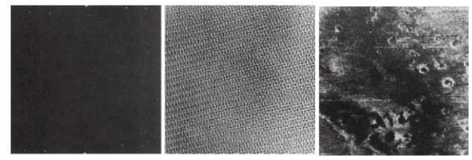
Example: Image Degraded by Periodic Noise

Degraded image

FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)



DFT (no shift)



DFT of noise | Noise | Restored image

52

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Notch Reject Filters

A notch reject filter is used to eliminate some frequency components.

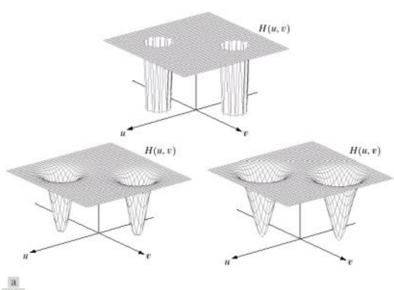


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

50

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Estimation of Degradation Model

Degradation model:

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

or

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

Purpose: to estimate $h(x,y)$ or $H(u,v)$

Why? If we know exactly $h(x,y)$, regardless of noise, we can do deconvolution to get $f(x,y)$ back from $g(x,y)$.

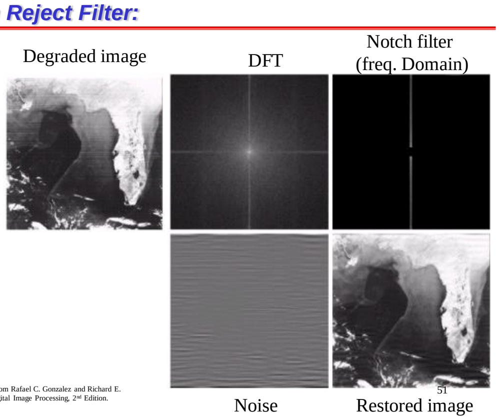
Methods:

1. Estimation by Image Observation
2. Estimation by Experiment
3. Estimation by Modeling

53

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

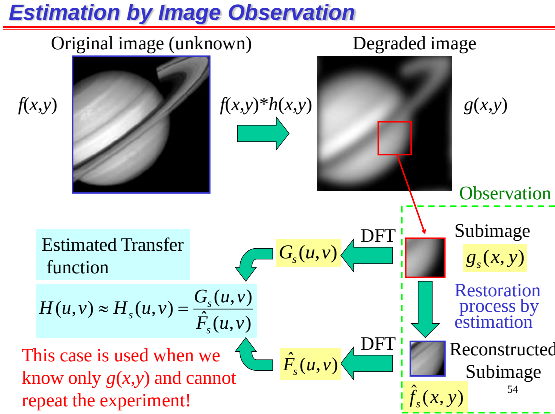
Notch Reject Filter:



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

51

Estimation by Image Observation

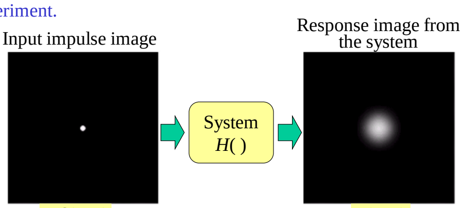


This case is used when we know only $g(x,y)$ and cannot repeat the experiment!

54

Estimation by Experiment

Used when we have the same equipment set up and can repeat the experiment.



$A\delta(x,y)$ DFT $DFT\{A\delta(x,y)\}=A$

$g(x,y)$ DFT $G(u,v)$

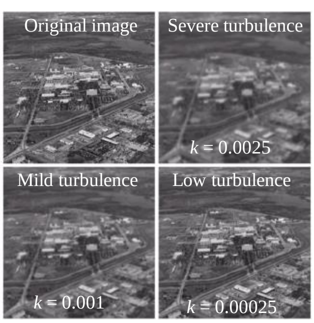
$$H(u,v)=\frac{G(u,v)}{A}$$

55

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Estimation by Modeling

Used when we know physical mechanism underlying the image formation process that can be expressed mathematically.



Example:

Atmospheric Turbulence model

$$H(u,v)=e^{-k(u^2+v^2)^{5/6}}$$

56

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Estimation by Modeling: Motion Blurring

Assume that camera velocity is $(x_0(t),y_0(t))$

The blurred image is obtained by

$$g(x,y)=\int_0^T f(x+x_0(t),y+y_0(t))dt$$

where T = exposure time.

$$G(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^T f(x+x_0(t),y+y_0(t))dt\right]e^{-j2\pi(ux+vy)}dxdy$$

$$=\int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x+x_0(t),y+y_0(t))e^{-j2\pi(ux+vy)}dxdy\right]dt$$

57

Estimation by Modeling: Motion Blurring (cont.)

$$G(u,v)=\int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x+x_0(t),y+y_0(t))e^{-j2\pi(ux+vy)}dxdy\right]dt$$

$$=\int_0^T\left[F(u,v)e^{-j2\pi(ux_0(t)+vy_0(t))}\right]dt$$

$$=F(u,v)\int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt$$

Then we get, the motion blurring transfer function:

$$H(u,v)=\int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt$$

For constant motion $(x_0(t),y_0(t))=(at,bt)$

$$H(u,v)=\int_0^T e^{-j2\pi(ua+vb)}dt=\frac{T}{\pi(ua+vb)}\sin(\pi(ua+vb))e^{-j\pi(ua+vb)}$$

58

Motion Blurring Example

For constant motion

$$H(u,v)=\frac{T}{\pi(ua+vb)}\sin(\pi(ua+vb))e^{-j\pi(ua+vb)}$$

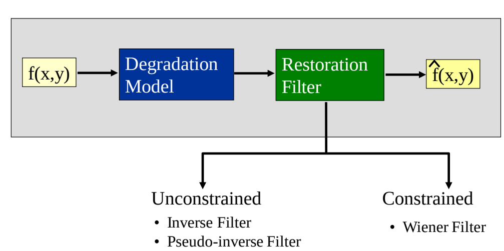
Original image

Motion blurred image $a=b=0.1$, $T=1$

59

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Restoration Model



demos/demo5blur_invfilter/

60

Inverse Filter

From degradation model:

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

after we obtain $H(u,v)$, we can estimate $F(u,v)$ by the inverse filter:

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Noise is enhanced when $H(u,v)$ is small.

To avoid the side effect of enhancing noise, we can apply this formulation to freq. component (u,v) with in a radius D_0 from the center of $H(u,v)$.

In practical, the inverse filter is not popularly used.

61

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.)

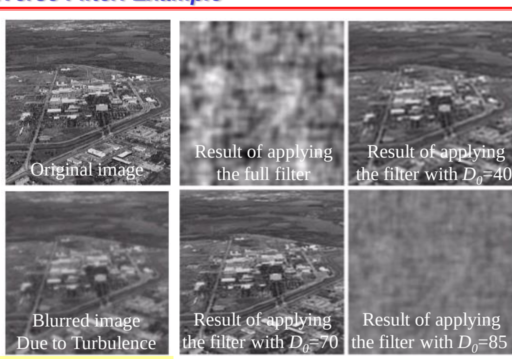
Inverse Filtering

Limitations:

- Even if the degradation function is known the undegraded image cannot be recovered exactly because N(u,v) is the random function which is not known.
- If the degradation function has '0' or small value the ratio easily dominates the estimate F(u,v)
- One approach to get rid of 0 or small value problem is to limit the filter frequency to the value near the origin.

62

Inverse Filter: Example



$$H(u,v) = e^{-0.0025(u^2+v^2)^{5/6}}$$

63

WIENER FILTERING

- Inverse filtering has no explicit provision for handling noise
- The wiener filter incorporates both degradation function, and statistical characteristics of noise in the restoration process.
- Objective of the wiener filter is to find the estimate of uncorrupted image f, such that the mean square error is minimum.
- The wiener filter is an optimum filter
- Conditions
  - (1) Noise and image are uncorrelated
  - (2) One or the other has zero mean
  - (3) Gray levels in $\hat{f}$ are linear function of gray levels in g

64

Norbert Wiener (1894-1964)

The renowned MIT professor Norbert Wiener was famed for his absent-mindedness. While crossing the MIT campus one day, he was stopped by a student with a mathematical problem. The perplexing question answered, Norbert followed with one of his own: "In which direction was I walking when you stopped me?" he asked, prompting an answer from the curious student. "Ah," Wiener declared, "then I've had my lunch"

Anecdote of Norbert Wiener

65

Wiener Filter: Minimum Mean Square Error Filter

Objective: optimize mean square error: $e^2 = E\{(f - \hat{f})^2\}$

Wiener Filter Formula:

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$$= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

where

$H(u,v)$ = Degradation function
$S_\eta(u,v)$ = Power spectrum of noise
$S_f(u,v)$ = Power spectrum of the undegraded image

66

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.)

Wiener Filter: Minimum Mean Square Error Filter…

- H(u, v) = degradation function
- H*(u, v) = complex conjugate of H(u, v)
- $|H(u, v)|^2 = H^*(u, v) H(u, v)$
- $S_\eta(u, v) = |N(u, v)|^2$ = power spectrum of the noise
- $S_f(u, v) = |F(u, v)|^2$ = power spectrum of undegraded image

- No problem with zeros unless H(u, v) and Sη(u, v) are both zero

- When noise is zero, Wiener filter = inverse filter

Approximation of Wiener Filter

Since $S\eta(u, v) = |N(u, v)|^2$ and $S_f(u, v) = |F(u, v)|^2$ are seldom known, the Wiener filter is frequently approximated

Wiener Filter Formula:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

Difficult to estimate

Approximated Formula:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

Practically, K is chosen manually to obtained the best visual result!

Wiener Filtering

ADVANTAGES:

1.The wiener filter does not have zero value problem.

2.The result obtained by wiener filter is more closer to the original image than inverse filter.

Image Example

motion blurred image

deblurred image after wiener filtering (K=0.01)

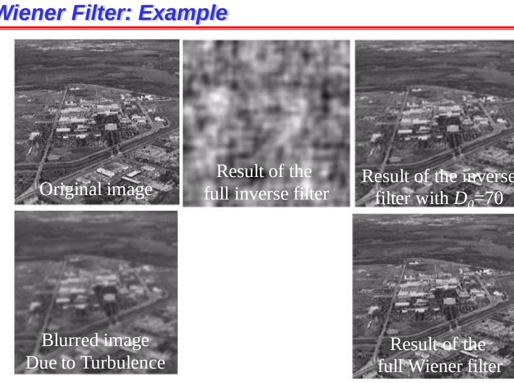
Image Example (Con't)

K=0.1

K=0.01

K=0.001

Wiener Filter: Example

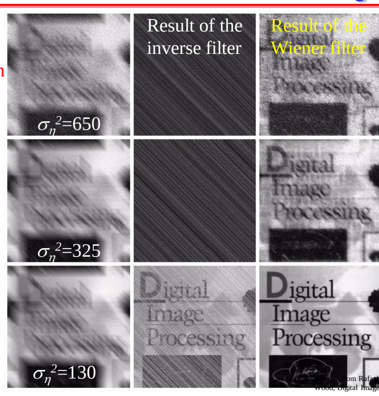


Wiener Filter: Example (cont.)

73

Example: Wiener Filter and Motion Blurring

Image degraded by motion blur + AWGN



Note: K is chosen manually

74

...om Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Constrained Least Squares Filter

Degradation model:

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

Written in a matrix form

$$\mathbf{g} = \mathbf{Hf} + \boldsymbol{\eta}$$

Objective: to find the minimum of a criterion function

$$C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\nabla^2 f(x,y)\right]^2$$

Subject to the constraint

$$\left\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\right\|^2 = \|\boldsymbol{\eta}\|^2$$

where $\|\mathbf{w}\|^2 = \mathbf{w}^T\mathbf{w}$

We get a constrained least square filter

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right]G(u,v)$$

where

$$P(u,v) = \text{Fourier transform of } p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

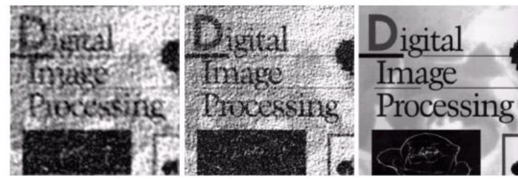
75

Constrained Least Squares Filter: Example

Constrained least square filter

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right]G(u,v)$$

γ is adaptively adjusted to achieve the best result.

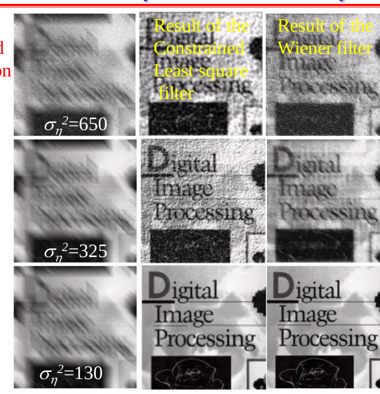


Results from the previous slide obtained from the constrained least square filter

76

Constrained Least Squares Filter: Example (cont.)

Image degraded by motion blur + AWGN



77

Constrained Least Squares Filter: Adjusting γ

Define $\mathbf{r} = \mathbf{g} - \mathbf{H}\hat{\mathbf{f}}$ It can be shown that $\phi(\gamma) = \mathbf{r}^T\mathbf{r} = \|\mathbf{r}\|^2$

We want to adjust gamma so that $\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2 \pm a$ → (1)

where a = accuracy factor

1. Specify an initial value of γ
2. Compute $\|\mathbf{r}\|^2$
3. Stop if (1) is satisfied
   Otherwise return step 2 after increasing γ if $\|\mathbf{r}\|^2 < \|\boldsymbol{\eta}\|^2 - a$
   or decreasing γ if $\|\mathbf{r}\|^2 > \|\boldsymbol{\eta}\|^2 + a$
   Use the new value of γ to recompute

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right]G(u,v)$$

78

Constrained Least Squares Filter: Adjusting γ (cont.)

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

$$R(u,v) = G(u,v) - H(u,v)\hat{F}(u,v)$$

$$\|\mathbf{r}\|^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} r^2(x,y)$$

For computing $\|\mathbf{r}\|^2$

$$m_\eta = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \eta(x,y)$$

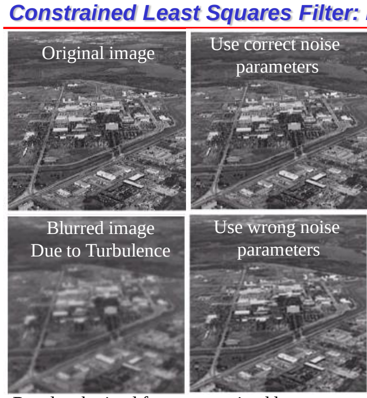
$$\sigma_\eta^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\eta(x,y) - m_\eta \right]^2$$

$$\|\boldsymbol{\eta}\|^2 = MN\left[\sigma_\eta^2 - m_\eta \right]$$

For computing $\|\boldsymbol{\eta}\|^2$

79

Constrained Least Squares Filter: Example



Correct parameters:
Initial $\gamma = 10^{-5}$
Correction factor = 10^{-6}
a = 0.25
$\sigma_\eta^2 = 10^{-5}$

Wrong noise parameter
$\sigma_\eta^2 = 10^{-2}$

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

80

Results obtained from constrained least square filters