CHAPTER-8

Spread – Spectrum Modulation

Introduction:

Initially developed for military applications during II world war, that was less sensitive to intentional interference or jamming by third parties.

Spread spectrum technology has blossomed into one of the fundamental building blocks in current and next-generation wireless systems

Problem of radio transmission

Narrow band can be wiped out due to interference

To disrupt the communication, the adversary needs to do two things,

(a) to detect that a transmission is taking place and

(b) to transmit a jamming signal which is designed to confuse the receiver.

.**Solution**

A spread spectrum system is therefore designed to make these tasks as difficult as possible.

Firstly, the transmitted signal should be difficult to detect by an adversary/jammer, i.e., the signal should have a low probability of intercept (LPI).

Secondly, the signal should be difficult to disturb with a jamming signal, i.e., the transmitted signal should possess an anti-jamming (AJ) property

Remedy

spread the narrow band signal into a broad band to protect against interference

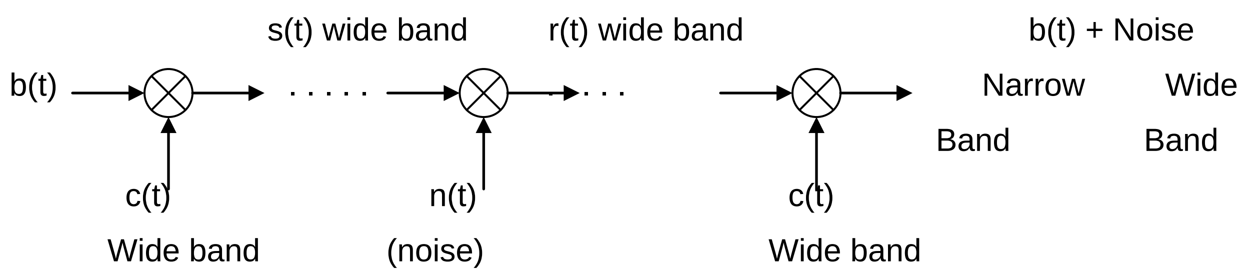
In a digital communication system the primary resources are **Bandwidth** and **Power.** The study of digital communication system deals with efficient utilization of these two resources, but there are situations where it is necessary to sacrifice their efficient utilization in order to meet certain other design objectives.

For example to provide a form of secure communication (i.e. the transmitted signal is not easily detected or recognized by unwanted listeners) the bandwidth of the transmitted signal is increased in excess of the minimum bandwidth necessary to transmit it. This requirement is catered by a technique known as **"Spread Spectrum Modulation".**

The primary advantage of a Spread – Spectrum communication system is its ability to reject ‘**Interference**’ whether it be the unintentional or the intentional interference.

The definition of Spread – Spectrum modulation may be stated in two parts.

1. Spread Spectrum is a mean of transmission in which the data sequence occupies a BW (Bandwidth) in excess of the minimum BW necessary to transmit it.
2. The Spectrum Spreading is accomplished before transmission through the use of a code that is independent of the data sequence. The Same code is used in the receiver to despread the received signal so that the original data sequence may be recovered.



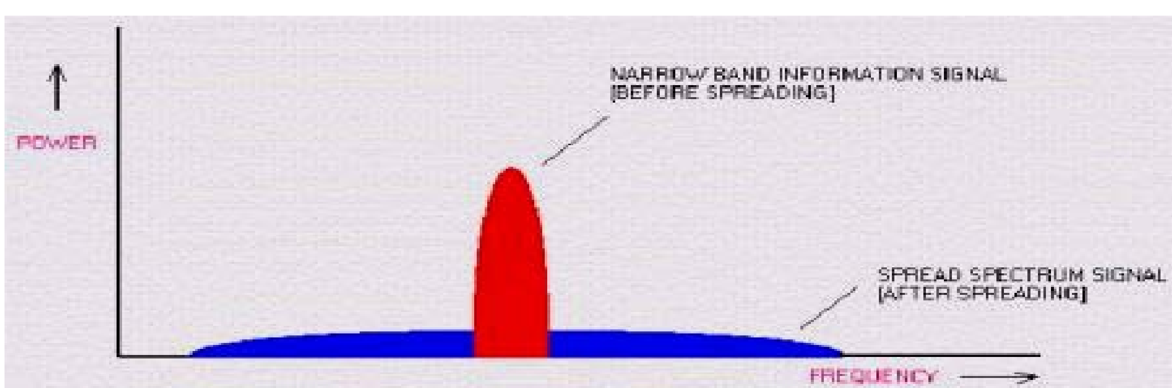
---- Transmitter---- ---- Channel------ --- Receiver--------

fig:1 spread spectrum technique.

b(t) = Data Sequence to be transmitted (Narrow Band)

c(t) = Wide Band code

s(t) = c(t) * b(t) – (wide Band)



<u>Fig:</u> Spectrum of signal before & after spreading

<u>PSUEDO-NOISE SEQUENCE</u>:

Generation of PN sequence:

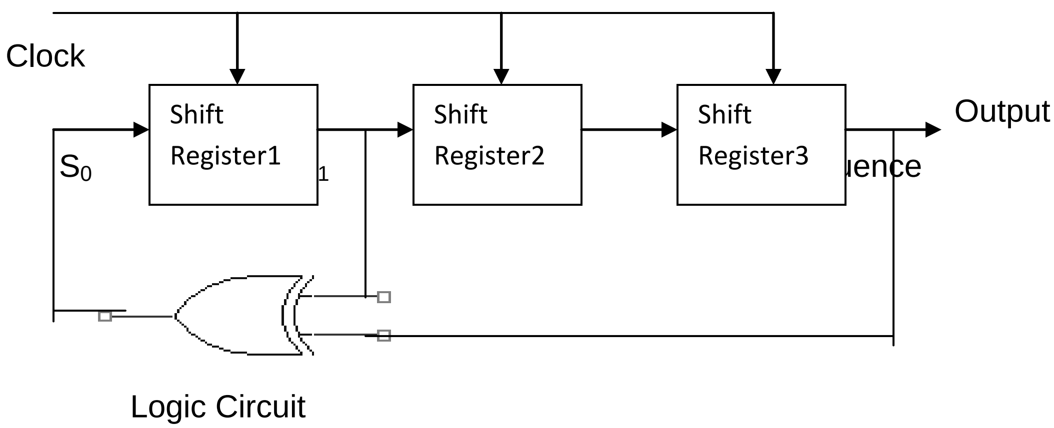


Fig 2: Maximum-length sequence generator for n=3

A feedback shift register is said to be <u>Linear</u> when the feed back logic consists of entirely mod-2-address (Ex-or gates). In such a case, the <u>zero state</u> is <u>not permitted.</u> The period of a PN sequence produced by a linear feedback shift register with ‘n’ flip flops cannot exceed 2^n-1. When the period is exactly 2^n-1, the PN sequence is called a ‘**<u>maximum length sequence’</u>** or ‘**<u>m-sequence’</u>**.

Example1: Consider the linear feed back shift register as shown in fig 2 involve three flip-flops. The input s_o is equal to the mod-2 sum of S_1 and S_3. If the initial state of the shift register is 100. Then the succession of states will be as follows.

100,110,011,011,101,010,001,100 . . . . . .

The output sequence (output S_3) is therefore. 00111010 . . . . .

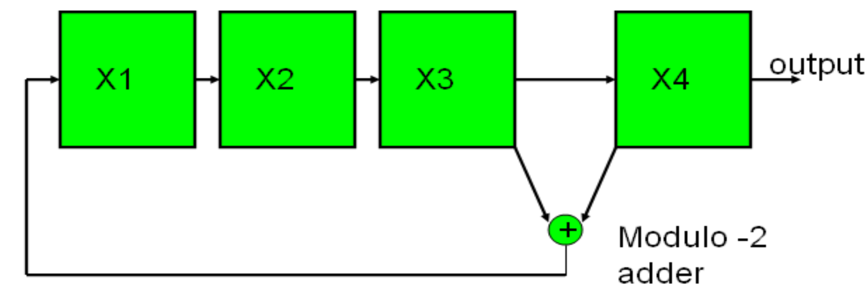
Which repeats itself with period 2^3–1 = 7 (n=3)

Maximal length codes are commonly used PN codes

In binary shift register, the maximum length sequence is

$$\mathbf{N = 2^m - 1}$$

chips, where **m** is the number of stages of flip-flops in the shift register.



Linear Feedback Shift Register with modulo two adder

At each clock pulse

- Contents of register shifts one bit right.
- Contents of required stages are modulo 2 added and fed back to input.

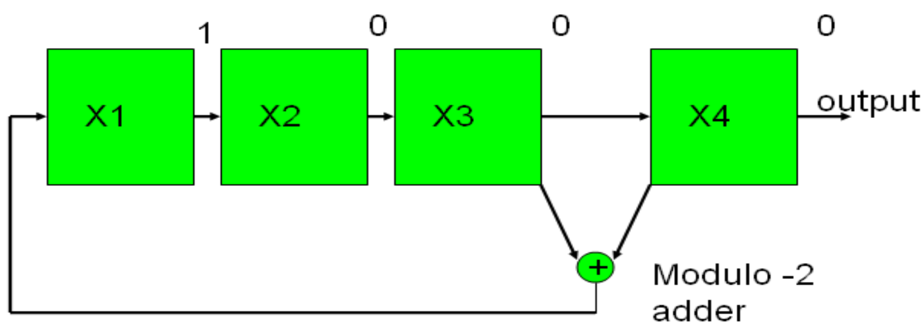


Fig: Initial stages of Shift registers **1000**

Let initial status of shift register be 1000

1	0	0	0
0	1	0	0
0	0	1	0
1	0	0	1
1	1	0	0
0	1	1	0
1	0	1	1
0	1	0	1
1	0	1	0
1	1	0	1

- We can see for shift Register of length m=4. .At each clock the change in state of flip-flop is shown.
- Feed back function is modulo two of X_3 and X_4.
- After 15 clock pulses the sequence repeats. Output sequence is

0 0 0 1 0 0 1 1 0 1 0 1 1 1 1

Properties of PN Sequence

Randomness of PN sequence is tested by following properties

1. Balance property
2. Run length property
3. Autocorrelation property

1. Balance property

In each Period of the sequence , number of binary ones differ from binary zeros by at most one digit .

Consider output of shift register 0 0 0 1 0 0 1 1 0 1 0 1 1 1 1
Seven zeros and eight ones -meets balance condition.

2. Run length property

Among the runs of ones and zeros in each period, it is desirable that about one half the runs of each type are of length 1, one- fourth are of length 2 and one-eighth are of length 3 and so-on.

Consider output of shift register

Number of runs =8

0 0 0 1 0 0 1 1 0 1 0 1 1 1 1

3 1 2 2 1 1 1 4

3. Auto correlation property

Auto correlation function of a maximal length sequence is periodic and binary valued.

Autocorrelation sequence of binary sequence in polar format is given by

$$R_c(k) = \frac{1}{N} \sum_{n=1}^{N} c_n c_{n-k}$$

Where **N** is length or period of the sequence and

k is the lag of the autocorrelation

$$R_c(k) = \begin{cases} \mathbf{1} & \text{if } k = lN \\ -\frac{1}{N} & k \neq lN \end{cases}$$

Where **l** is any integer.
we can also state Autocorrelation function as $\mathbf{R_c(k)} = \frac{\mathbf{1}}{\mathbf{N}}$

{ No. of agreements – No. of disagreements in comparison of one full period }
Consider output of shift register for l=1

0 0 0 1 0 0 1 1 0 1 0 1 1 1 1

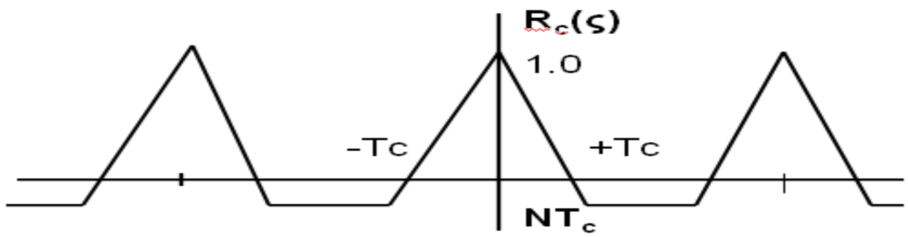
1 0 0 0 1 0 0 1 1 0 1 0 1 1 1

d a a d d a d a d a d d a a a

$$R_c(k) = \frac{1}{15}(7-8)$$

$$R_c(k) = -\frac{1}{15}$$

Yields PN autocorrelation as



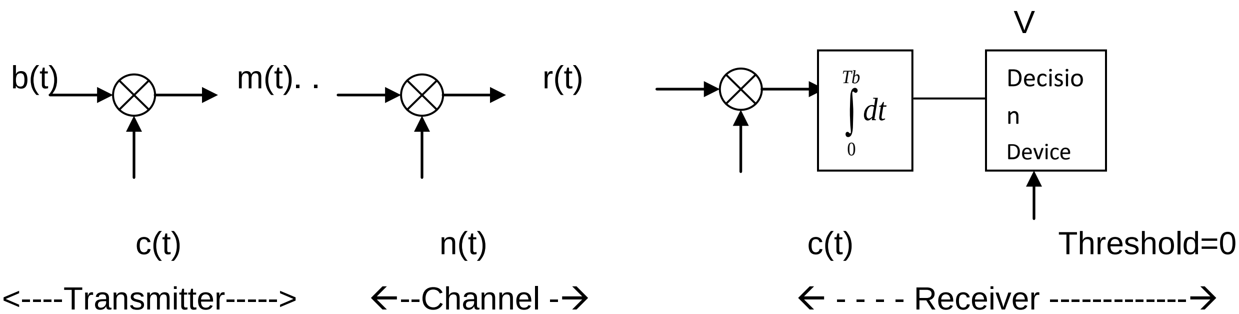
PN autocorrelation function.

Range of PN Sequence Lengths

Length 0f Shift Register, m	PN Sequence Length,
7	127
8	255
9	511
10	1023
11	2047
12	4095
13	8191
17	131071
19	524287

A Notion of Spread Spectrum:

An important attribute of Spread Spectrum modulation is that it can provide protection against externally generated interfacing signals with finite power. Protection against jamming (interfacing) waveforms is provided by purposely making the information – bearing signal occupy a BW far in excess of the minimum BW necessary to transmit it. This has the effect of making the transmitted signal a noise like appearance so as to blend into the background. Therefore Spread Spectrum is a method of 'camouflaging' the information – bearing signal.



Let { b_K} denotes a binary data sequence.

{ c_K } denotes a PN sequence.

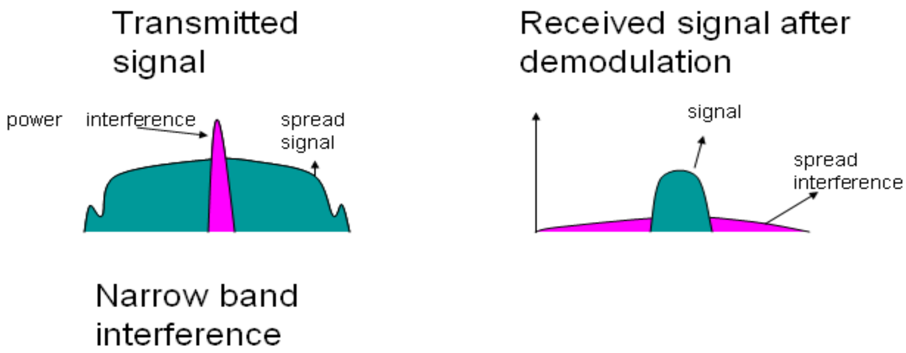
b(t) and c(t) denotes their NRZ polar representation respectively.

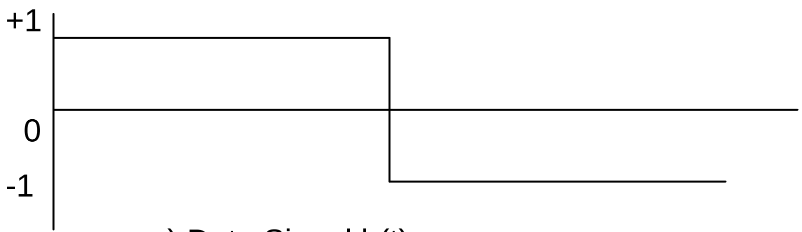
The desired modulation is achieved by applying the data signal b(t) and PN signal c(t) to a product modulator or multiplier. If the message signal b(t) is narrowband and the PN sequence signal c(t) is wide band, the product signal m(t) is also wide band. The PN sequence performs the role of a '**Spreading Code**".

For base band transmission, the product signal m(t) represents the transmitted signal. Therefore $m(t) = c(t).b(t)$

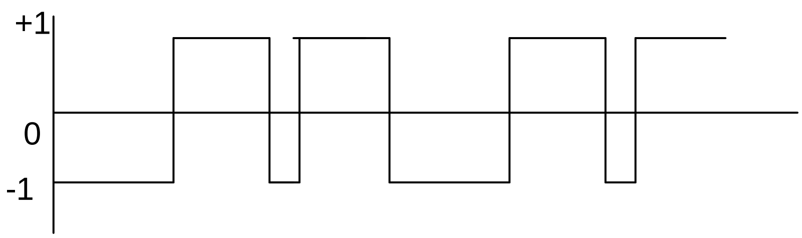
The received signal r(t) consists of the transmitted signal m(t) plus an additive interference noise n(t), Hence

$$r(t) = m(t) + n(t)$$
$$= c(t).b(t) + n(t)$$

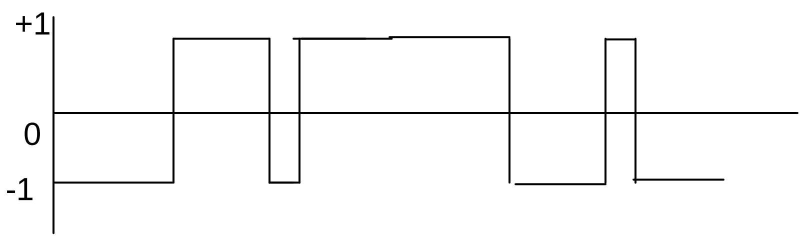




a) Data Signal b(t)



b)Spreading Code c(t)



c)Product signal or base band transmitted signal m(t)

To recover the original message signal b(t), the received signal r(t) is applied to a demodulator that consists of a multiplier followed by an integrator and a decision device. The multiplier is supplied with a locally generated PN sequence that is exact replica of that used in the transmitter. The multiplier output is given by

$Z(t) = r(t).c(t)$

$= [b(t) * c(t) + n(t)]\ c(t)$

$= c^2(t).b(t) + c(t).n(t)$

The data signal b(t) is multiplied twice by the PN signal c(t), where as unwanted signal n(t) is multiplied only once. But $c^2(t) = 1$, hence the above equation reduces to

$Z(t) = b(t) + c(t).n(t)$

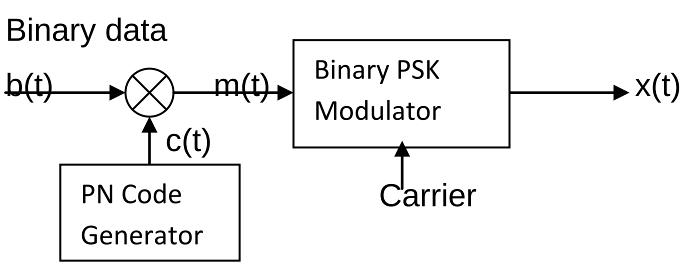
Now the data component b(t) is narrowband, where as the spurious component c(t)n(t) is wide band. Hence by applying the multiplier output to a base band (low pass) filter most of the power in the spurious component c(t)n(t) is filtered out. Thus the effect of the interference n(t) is thus significantly reduced at the receiver output.

The integration is carried out for the bit interval $0 \leq t \leq T_b$ to provide the sample value V. Finally, a decision is made by the receiver.

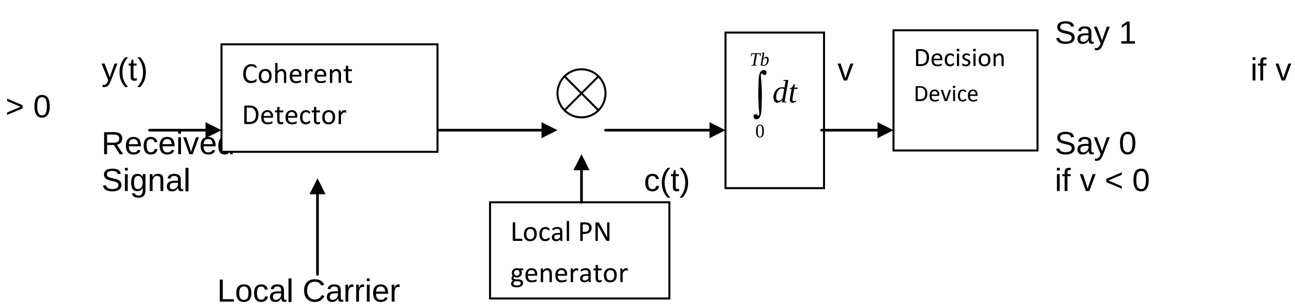
If V > Threshold Value '0', say binary symbol '1'

If V < Threshold Value '0', say binary symbol '0'

<u>Direct – Sequence Spread Spectrum with coherent binary Phase shift Keying:-</u>



a) Transmitter



b) Receiver

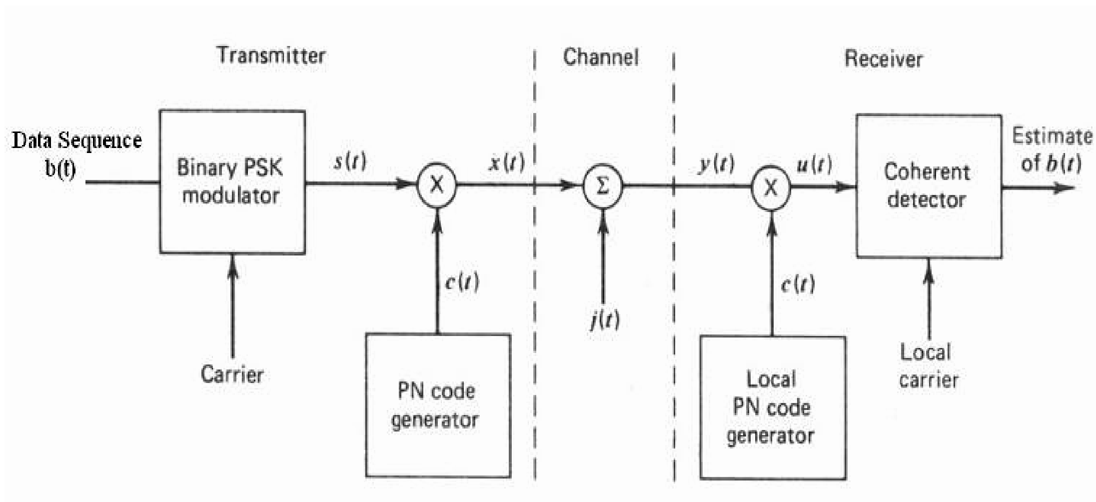


Fig: model of direct – sequence spread binary PSK system(alternative form)

To provide band pass transmission, the base band data sequence is multiplied by a Carrier by means of shift keying. Normally binary phase shift keying (PSK) is used because of its advantages.

The transmitter first converts the incoming binary data sequence $\{b_k\}$ into an NRZ waveform b(t), which is followed by two stages of modulation.

The first stage consists of a multiplier with data signal b(t) and the PN signal c(t) as inputs. The output of multiplier is m(t) is a wideband signal. Thus a narrow – band data sequence is transformed into a noise like wide band signal.

The second stage consists of a binary Phase Shift Keying (PSK) modulator. Which converts base band signal m(t) into band pass signal x(t). The transmitted signal x(t) is thus a direct – sequence spread binary PSK signal. The phase modulation θ(t) of x(t) has one of the two values '0' and 'π' (180^o) depending upon the polarity of the message signal b(t) and PN signal c(t) at time t.

Polarity of PN & Polarity of PN signal both +, + or - - Phase '0'

Polarity of PN & Polarity of PN signal both +, - or - + Phase 'π'

		Polarity of data sequence b(t)	
		+	-
Polarity of PN sequence C(t)	+	0	

			π
	-	π	0

The receiver consists of two stages of demodulation.

In the first stage the received signal y(t) and a locally generated carrier are applied to a coherent detector (a product modulator followed by a low pass filter), Which converts band pass signal into base band signal.

The second stage of demodulation performs Spectrum despreading by multiplying the output of low-pass filter by a locally generated replica of the PN signal c(t), followed by integration over a bit interval T_b and finally a decision device is used to get binary sequence.

Signal Space Dimensionality and Processing Gain

- Fundamental issue in SS systems is how much protection spreading can provide against interference.
- SS technique distribute low dimensional signal into large dimensional signal space (hide the signal).
- Jammer has only one option; to jam the entire space with fixed total power or to jam portion of signal space with large power.

Consider set of orthonormal basis functions;

$$\varphi_k(t) = \begin{cases} \sqrt{\frac{2}{T_c}}\cos(2\pi f_c t) & kT_c \le t \le (k+1)T_c \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{\varphi}_k(t) = \begin{cases} \sqrt{\frac{2}{T_c}}\sin(2\pi f_c t) & kT_c \le t \le (k+1)T_c \\ 0 & \text{otherwise} \end{cases} \qquad k = 0,1,\ldots\ldots\ldots\ldots,N-1$$

where
T_c is chip duration,
N is number of chips per bit.

Transmitted signal x(t) for the interval of an information bit is

$$\begin{aligned} x(t) &= c(t)\, s(t) \\ &= \pm \sqrt{\frac{2E_b}{T_b}}\, c(t) \cos(2\pi f_c t) \\ &= \pm \sqrt{\frac{E_b}{N}} \sum_{k=0}^{N-1} c_k \varphi_k(t) \qquad 0 \le t \le T_b \end{aligned}$$

where, Eb is signal energy per bit.

PN Code sequence { c_0, c_1, ……c_{N-1}} with ck= $\pm$ 1

Transmitted signal x(t) is therefore N dimensional and requires N orthonormal functions to represent it.

j(t) represent interfering signal (jammer). As said jammer tries to places all its available energy in exactly same N dimension signal space. But jammer has no knowledge of signal phase. Hence tries to place equal energy in two phase coordinates that is cosine and sine

As per that jammer can be represented as

$$j(t) = \sum_{k=0}^{N-1} j_k \varphi_k(t) + \sum_{k=0}^{N-1} \tilde{j}_k \tilde{\varphi}_k(t) \qquad 0 \le t \le T_b$$

where

$$j_k = \int_0^{T_b} j(t) \varphi_k(t)\, dt \qquad k = 0,1,\ldots\ldots\ldots\ldots N-1$$

$$\tilde{j}_k = \int_0^{T_b} j(t) \tilde{\varphi}_k(t)\, dt \qquad k = 0,1,\ldots\ldots\ldots\ldots N-1$$

Thus j(t) is 2N dimensional, twice the dimension as that of x(t).

Average interference power of j(t)

$$J = \frac{1}{T_b} \int_0^{T_b} j^2(t)\, dt$$

$$= \frac{1}{T_b} \sum_{k=0}^{N-1} j_k^2 + \frac{1}{T_b} \sum_{k=0}^{N-1} \tilde{j}_k^2$$

as jammer places equal energy in two phase coordinates , hence

$$\sum_{k=0}^{N-1} j_k^2 = \sum_{k=0}^{N-1} \tilde{j}_k^2$$

$$J = \frac{2}{T_b} \sum_{k=0}^{N-1} j_k^2$$

To evaluate system performance we calculate SNR at input and output of **DS/BPSK** receiver.
The coherent receiver input is $u(t) = s(t) + c(t)j(t)$
and using this $u(t)$, output at coherent receiver

$$v = \sqrt{\frac{2}{T_b}} \int_0^{T_b} u(t) \cos(2\pi f_c t)\,dt$$
$$= v_s + v_{cj}$$

Where v_s is despread component of BPSK and v_{cj} of spread interference.

$$v_s = \sqrt{\frac{2}{T_b}} \int_0^{T_b} s(t) \cos(2\pi f_c t)\,dt$$

$$v_{cj} = \sqrt{\frac{2}{T_b}} \int_0^{T_b} c(t)\, j(t) \cos(2\pi f_c t)\,dt$$

Consider despread BPSK signal s(t)

$$s(t) = \pm\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad 0 \le t \le T_b$$

Where + sign is for symbol 1
- sign for symbol 0.

If carrier frequency is integer multiple of $1 / T_b$, wehave

$$v_s = \pm\sqrt{E_b}$$

Consider spread interference component v_{cj}
here c(t) is considered in sequence form $\{ c_0, c_1, \ldots\ldots c_{N-1}\}$

$$v_{cj} = \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} c_k \int_0^{T_b} j(t) \varphi_k(t)\,dt$$

$$= \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} c_k\, j_k$$

With C_k treated as independent identical random variables with both symbols having equal probabilities

$$P(C_k = 1) = P(C_k = -1) = \frac{1}{2}$$

Expected value of Random variable v_{cj} is zero, for fixed **k** we have

$$\begin{aligned} E\left[C_k j_k \middle| j_k\right] &= j_k P(C_k = 1) - j_k P(C_k = -1) \\ &= \frac{1}{2} j_k - \frac{1}{2} j_k \\ &= 0 \end{aligned}$$

and Variance

$$Var\left[V_{cj} \middle| j\right] = \frac{1}{N} \sum_{k=0}^{N-1} j_k^2 = \frac{JT_c}{2}$$

Spread factor $\mathbf{N = T_b/T_c}$
Output signal to noise ratio is

$$(SNR)_0 = \frac{2E_b}{JT_c}$$

The average signal power at receiver input is E_b/T_b hence input SNR

$$(SNR)_I = \frac{E_b/T_b}{J}$$

$$(SNR)_0 = \frac{2T_b}{T_c}(SNR)_I$$

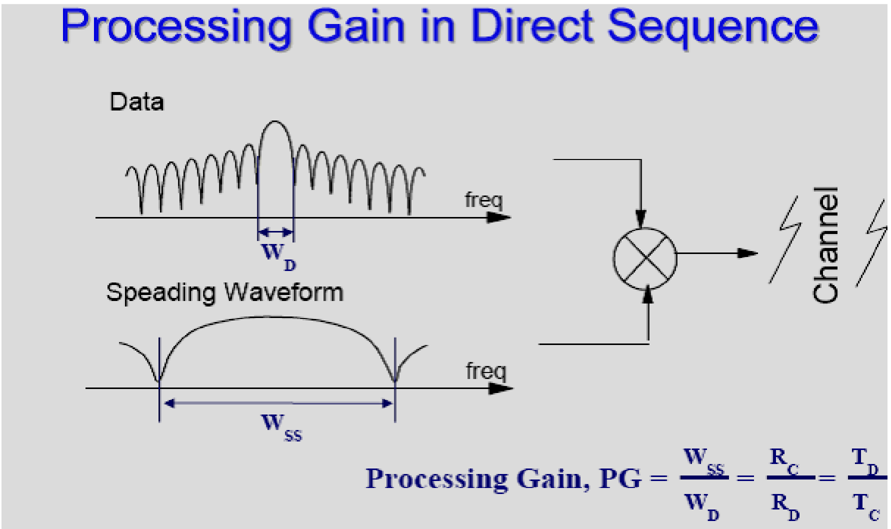
Expressing SNR in decibels

$$10\log_{10}(SNR)_0 = 10\log_{10}(SNR)_I + 3 + 10\log_{10}(PG), dB$$

where $$PG = \frac{T_b}{T_c}$$

- .3db term on right side accounts for gain in SNR due to coherent detection.
- . Last term accounts for gain in SNR by use of spread spectrum.

PG is called Processing Gain



1. Bit rate of binary data entering the transmitter input is

$$R_b = \frac{1}{T_b}$$

2. The bandwidth of PN sequence c(t) , of main lobe is W_c

$$W_c = \frac{1}{T_c}$$

$$PG = \frac{W_c}{R_b}$$

<u>Probability of error</u>

To calculate probability of error, we consider output component **v** of coherent detector as sample value of random variable **V**

$$V = \pm\sqrt{E_b} + V_{cj}$$

E_b is signal energy per bit and V_{cj} is noise component

Decision rule is, if detector output exceeds a threshold of zero volts; received bit is symbol 1 else decision is favored for zero.

- Average probability of error P_e is nothing but conditional probability which depends on random variable V_{cj}.

- As a result receiver makes decision in favor of symbol 1 when symbol 0 transmitted and vice versa
- Random variable Vcj is sum of N such random variables. Hence for

Large N it can assume Gaussian distribution .
- As mean and variance has already been discussed , zero mean and variance **$JT_c/2$**

Probability of error can be calculated from simple formula for DS/BPSK system

$$P_e \cong \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right)$$

<u>Antijam Characteristics</u>

Consider error probability of BPSK

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Comparing both probabilities;

$$\frac{N_0}{2} = \frac{JT_c}{2}$$

Since bit energy $E_b = PT_b$, P= average signal power.

We can express bit energy to noise density ratio as

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P}{J}\right)$$

or

$$\frac{J}{P} = \frac{PG}{E_b/N_0}$$

The ratio **J/P** is termed jamming margin. Jamming Margin is expressed in decibels as

$$(\text{jamming margin})_{dB} = (\text{Processing gain})_{dB} - 10\log_{10}\left(\frac{Eb}{N_0}\right)_{min}$$

$$\left[\frac{Eb}{N_0}\right]$$

Where is minimum bit energy to noise ratio needed to support a prescribed average probability of error.

Example1

A pseudo random sequence is generated using a feed back shift register of length m=4. The chip rate is 107 chips per second. Find the following

a) **PN sequence length**
b) **Chip duration of PN sequence**
c) **PN sequence period**

Solution

a) Length of PN sequence $N = 2^m - 1$
$= 2^4 - 1 = 15$

b) Chip duration T_c = 1/chip rate =1/107 = 0.1µsec

c) PN sequence period $T = NT_c$
= 15 x 0.1µsec = 1.5µsec

Example2

A direct sequence spread binary phase shift keying system uses a feedback shift register of length 19 for the generation of PN sequence. Calculate the processing gain of the system.

Solution

Given length of shift register = m =19
Therefore length of PN sequence $N = 2^m - 1$
$= 2^{19} - 1$

Processing gain $PG = T_b/T_c = N$
in db $= 10\log_{10}N = 10\log_{10}(2^{19})$
= 57db

Example3

A Spread spectrum communication system has the following parameters. Information bit duration Tb = 1.024 msecs and PN chip duration of 1µsecs. The average probability of error of system is not to exceed 10^{-5}. calculate a) Length of shift register b) Processing gain c) jamming margin

Solution

Processing gain PG =N= Tb/Tc =1024 corresponding length of shift register m = 10
In case of coherent BPSK For Probability of error 10^{-5}.
[Referring to error function table]
$E_b/N_0 = 10.8$
Therefore jamming margin

$$\textbf{(jammingmargin)}_{\textbf{dB}} = \textbf{(Processinggain)}_{\textbf{dB}} - \textbf{10log}_{\textbf{10}}\left(\frac{\textbf{Eb}}{\textbf{N}_0}\right)_{\textbf{min}}$$

$$(\text{jamming margin})_{\text{dB}} = 10\log_{10}\text{PG}_{\text{dB}} - 10\log_{10}\left(\frac{\text{Eb}}{\text{N}_0}\right)_{\min}$$

$$= 10\log_{10}1024 - 10\log_{10}10.8$$

= 30.10 – 10.33
= 19.8 db

Frequency – Hop Spread Spectrum:

In a frequency – hop Spread – Spectrum technique, the spectrum of data modulated carrier is widened by changing the carrier frequency in a pseudo – random manner. The type of spread – spectrum in which the carrier hops randomly form one frequency to another is called **Frequency – Hop (FH) Spread Spectrum.**

Since frequency hopping does not covers the entire spread spectrum instantaneously. We are led to consider the rate at which the hop occurs. Depending upon this we have two types of frequency hop.

1. Slow frequency hopping:- In which the symbol rate R_s of the MFSK signal is an integer multiple of the hop rate R_h. That is several symbols are transmitted on each frequency hop.
2. Fast – Frequency hopping:- In which the hop rate R_h is an integral multiple of the MFSK symbol rate R_s. That is the carrier frequency will hoop several times during the transmission of one symbol.

   A common modulation format for frequency hopping system is that of M- ary frequency – shift – keying (MFSK).

Slow frequency hopping:-

Fig.a) Shows the block diagram of an FH / MFSK transmitter, which involves frequency modulation followed by mixing.

The incoming binary data are applied to an M-ary FSK modulator. The resulting modulated wave and the output from a digital frequency synthesizer are then applied to a mixer that consists of a multiplier followed by a band – pass filter. The filter is designed to select the sum frequency component resulting from the multiplication process as the transmitted signal. An 'k' bit segments of a PN sequence drive the frequency synthesizer, which enables the carrier frequency to hop over 2^n distinct values. Since frequency synthesizers are unable to maintain phase coherence over successive hops, most frequency hops spread spectrum communication system use non coherent M-ary modulation system.

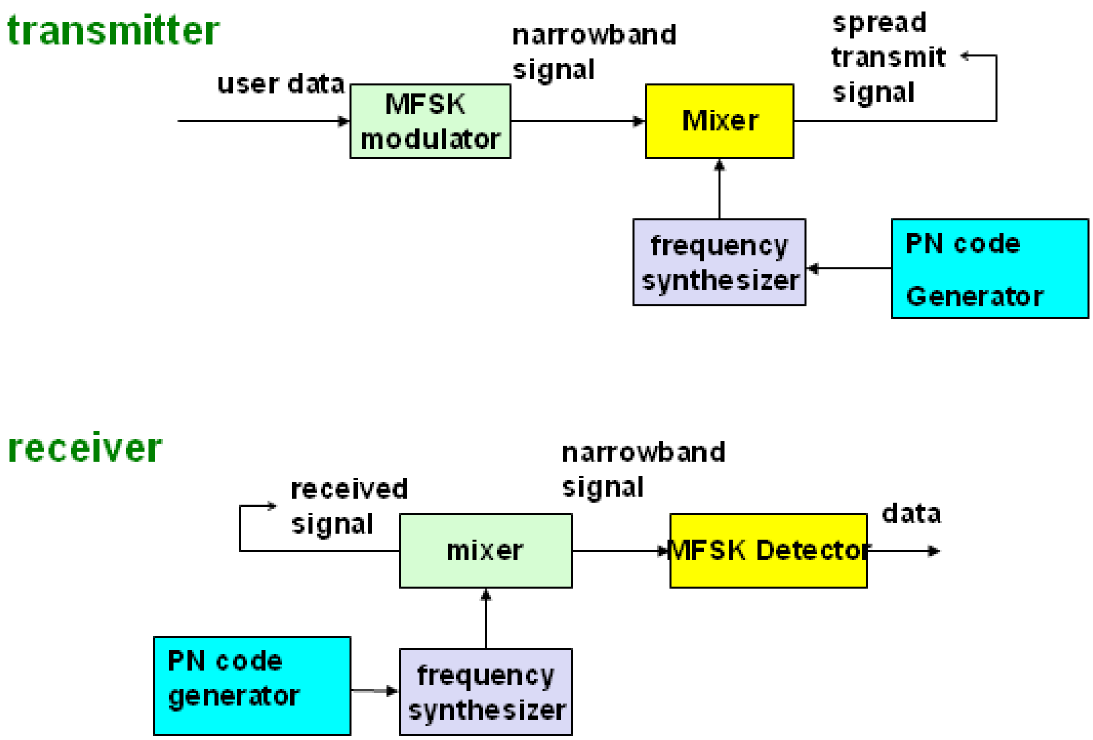


Fig a :- Frequency hop spread M-ary Frequency – shift – keying

In the receiver the frequency hoping is first removed by mixing the received signal with the output of a local frequency synthesizer that is synchronized with the transmitter. The resulting output is then band pass filtered and subsequently processed by a non coherent M-ary FSK demodulator. To implement this M-ary detector, a bank of M non coherent matched filters, each of which is matched to one of the MFSK tones is used. By selecting the largest filtered output, the original transmitted signal is estimated.

An individual FH / MFSK tone of shortest duration is referred as a chip. The chip rate R_c for an FH / MFSK system is defined by

$R_c = Max(R_h, R_s)$

Where R_h is the hop rate and R_s is Symbol Rate

In a slow rate frequency hopping multiple symbols are transmitted per hop. Hence each symbol of a slow FH / MFSK signal is a chip. The bit rate R_b of the incoming binary data. The symbol rate R_s of the MFSK signal, the chip rate R_c and the hop rate R_n are related by

$$R_c = R_s = R_b / k \geq R_h$$

where $k = \log_2 M$

Fast frequency hopping:-

A fast FH / MFSK system differs from a slow FH / MFSK system in that there are multiple hops per m-ary symbol. Hence in a fast FH / MFSK system each hop is a chip.

Fast Frequency Hopping	**Slow Frequency Hopping**
Several frequency hops Per modulation	**Several modulation symbols per hop**
Shortest uninterrupted waveform in the system is that of hop	**Shortest uninterrupted waveform in the system is that of data symbol**
Chip duration =hop duration	**Chip duration=bit duration.**

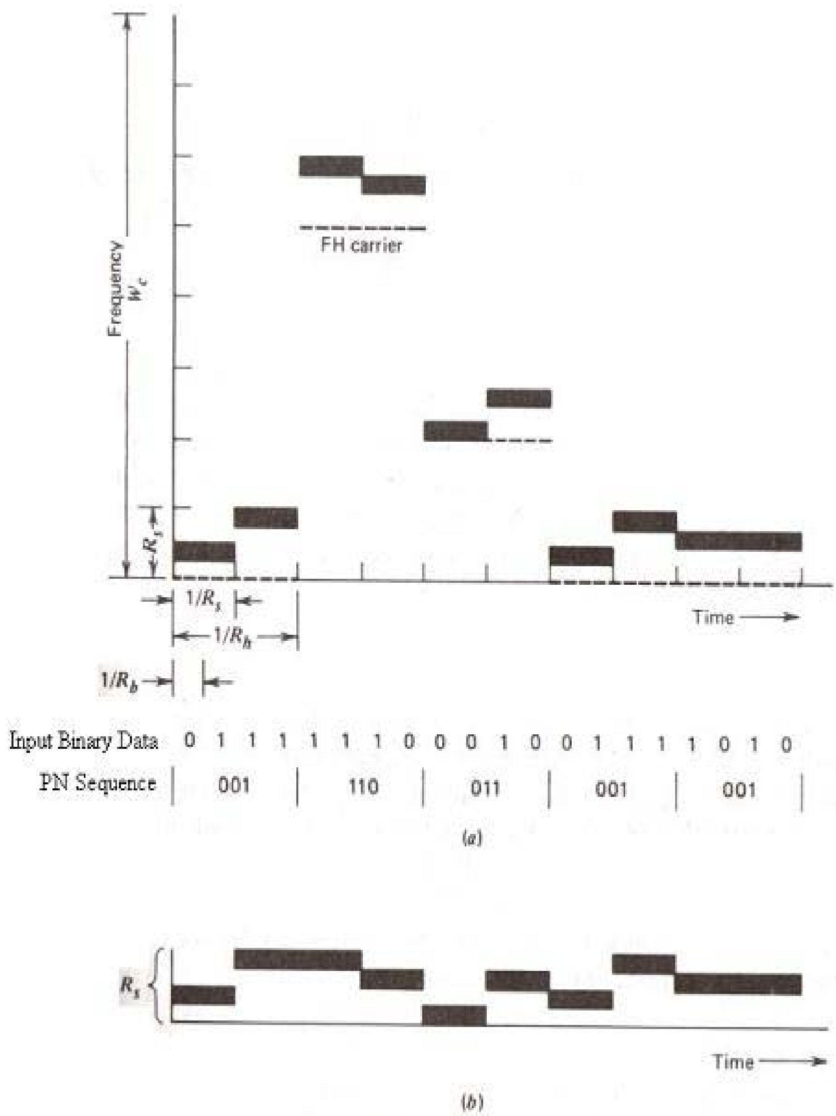
Fig. illustrates the variation of the frequency of a slow FH/MFSK signal with time for one complete period of the PN sequence. The period of the PN sequence is $2^4-1 = 15$. The FH/MFSK signal has the following parameters:

Number of bits per MFSK symbol $K = 2$.

Number of MFSK tones $M = 2^K = 4$

Length of PN segment per hop $k = 3$

Total number of frequency hops $2^k = 8$



Illustrating slow-frequency hopping.

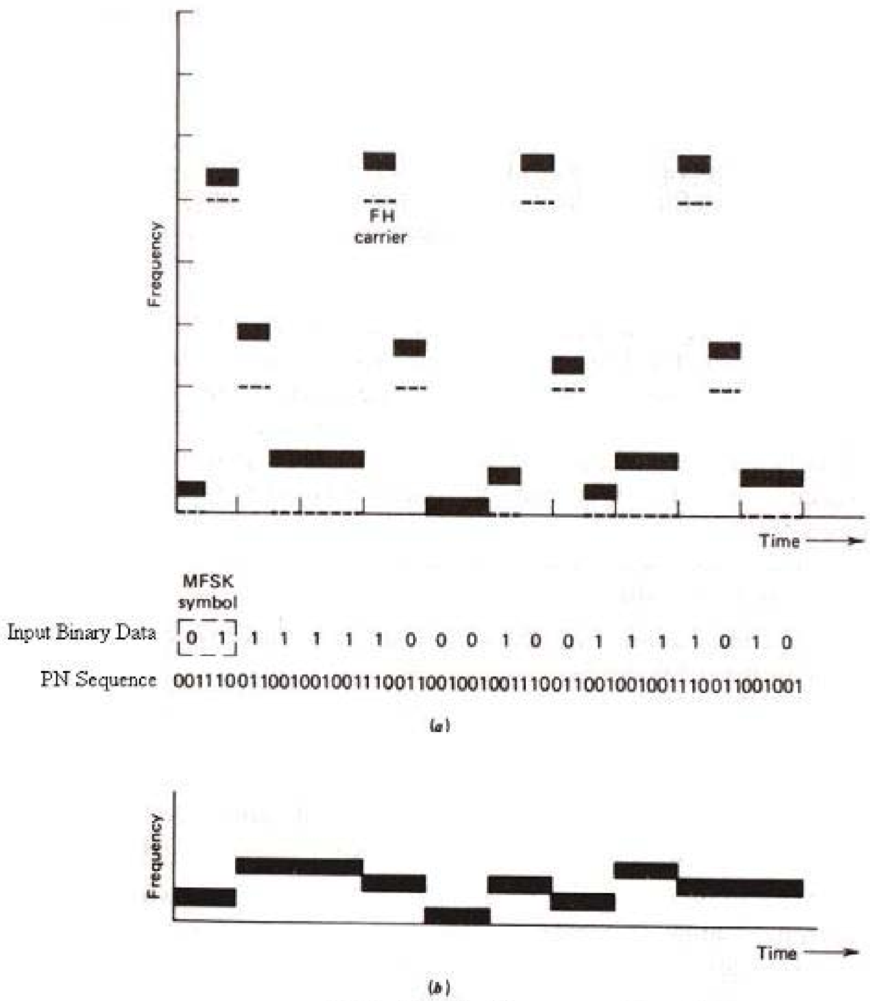
Fig. illustrates the variation of the transmitted frequency of a fast FH/MFSK signal with time. The signal has the following parameters:

Number of bits per MFSK symbol $K = 2$.

Number of MFSK tones $M = 2^K = 4$

Length of PN segment per hop $k = 3$

Total number of frequency hops $2^k = 8$



Illustrating fast-frequency hopping.