CRYPTOGRAPHY AND NETWORK SECURITY
UNIT 2

UNIT 2
CLASSICAL ENCRYPTION TECHNIQUES

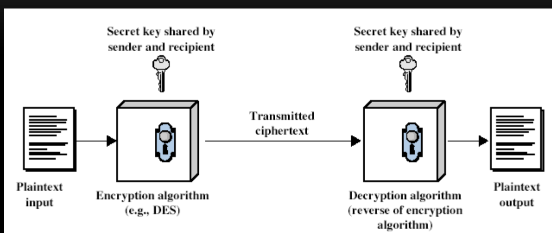
SYMMETRIC ENCRYPTION

- or conventional / private-key / single-key
- sender and recipient share a common key
- all classical encryption algorithms are private-key
- was only type prior to invention of public-key in 1970's
- and by far most widely used

SOME BASIC TERMINOLOGY

- **plaintext** - original message
- **ciphertext** - coded message
- **cipher** - algorithm for transforming plaintext to ciphertext
- **key** - info used in cipher known only to sender/receiver
- **encipher (encrypt)** - converting plaintext to ciphertext
- **decipher (decrypt)** - recovering ciphertext from plaintext
- **cryptography** - study of encryption principles/methods
- **cryptanalysis (codebreaking)** - study of principles/ methods of deciphering ciphertext *without* knowing key
- **cryptology** - field of both cryptography and cryptanalysis

SYMMETRIC CIPHER MODEL



SYMMETRIC CIPHER MODEL...

- plaintext - original message
- encryption algorithm – performs substitutions/transformations on plaintext
- secret key – control exact substitutions/transformations used in encryption algorithm
- ciphertext - scrambled message
- decryption algorithm – inverse of encryption algorithm

REQUIREMENTS

- two requirements for secure use of symmetric encryption:
  - a strong encryption algorithm
  - a secret key known only to sender / receiver
  - We assume that it is impractical to decrypt a message on the basis of the cipher- text plus knowledge of the cryption/decryption algorithm, and do not need to keep the algorithm secret; rather we only need to keep the key secret.
  - This feature of symmetric encryption is what makes it feasible for widespread use.
  - It allows easy distribution of s/w and h/w implementations.

REQUIREMENTS...

mathematically it can be considered a pair of functions with:

plaintext X, ciphertext Y, key K, encryption algorithm E_K, decryption algorithm D_K.

$Y = E_K(X)$

$X = D_K(Y)$

- assume encryption algorithm is known
- implies a secure channel to distribute key

CRYPTOGRAPHY

characterize cryptographic system by:

- Type of encryption operations used
  - substitution / transposition / product
- Number of keys used
  - single-key or private / two-key or public
- Way in which plaintext is processed
  - block / stream

CRYPTANALYSIS

- Objective is to recover key not just message
- General approaches:
  - **Cryptanalytic attack**
    - Attacks rely on the nature of the algorithm plus perhaps some knowledge of the general characteristics of the plaintext or even some sample plaintext-ciphertext pairs.
  - **Brute-force attack**
    - try every possible key on a piece of ciphertext until an intelligible translation into plaintext is obtained.
    - On average, half of all possible keys must be tried to achieve success

CRYPTANALYTIC ATTACKS

- **Ciphertext only**
  - Only know algorithm & ciphertext, is statistical, know or can identify plaintext
- **Known plaintext**
  - Know/suspect plaintext & ciphertext
- **Chosen plaintext**
  - Select plaintext and obtain ciphertext
- **Chosen ciphertext**
  - Select ciphertext and obtain plaintext
- **Chosen text**
  - Select plaintext or ciphertext to en/decrypt

MORE DEFINITIONS

- **Unconditional security**
  - No matter how much computer power or time is available, the cipher cannot be broken since the ciphertext provides insufficient information to uniquely determine the corresponding plaintext
- **Computational security**
  - Given limited computing resources (eg time needed for calculations is greater than age of universe), the cipher cannot be broken

BRUTE FORCE SEARCH

- Always possible to simply try every key
- Most basic attack, proportional to key size
- Assume either know / recognise plaintext

Key Size (bits)	Number of Alternative Keys	Time required at 1 decryption/µs	Time required at 10^6 decryptions/µs
32	$2^{32} = 4.3 \times 10^9$	2^{31} µs = 35.8 minutes	2.15 milliseconds
56	$2^{56} = 7.2 \times 10^{16}$	2^{55} µs = 1142 years	10.01 hours
128	$2^{128} = 3.4 \times 10^{38}$	2^{127} µs = 5.4×10^{24} years	5.4×10^{18} years
168	$2^{168} = 3.7 \times 10^{50}$	2^{167} µs = 5.9×10^{36} years	5.9×10^{30} years
26 characters (permutation)	$26! = 4 \times 10^{26}$	2×10^{26} µs = 6.4×10^{12} years	6.4×10^6 years

CLASSICAL SUBSTITUTION CIPHERS

- Where letters of plaintext are replaced by other letters or by numbers or symbols
- Or if plaintext is viewed as a sequence of bits, then substitution involves replacing plaintext bit patterns with ciphertext bit patterns

CAESAR CIPHER

- Earliest known substitution cipher
- By Julius Caesar
- First attested use in military affairs
- Replaces each letter by 3rd letter on
- Example:

  meet me after the toga party

  PHHW PH DIWHU WKH WRJD SDUWB

CAESAR CIPHER

- can define transformation as:

  a b c d e f g h i j k l m n o p q r s t u v w x y z

  D E F G H I J K L M N O P Q R S T U V W X Y Z A B C
- mathematically give each letter a number

  a b c d e f g h i j k l m n o p q r s t u v w x y z

  0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
- then have Caesar cipher as:

  $c = E(p) = (p + k) \bmod (26)$

  $p = D(c) = (c - k) \bmod (26)$

CRYPTANALYSIS OF CAESAR CIPHER

- only have 26 possible ciphers
  - A maps to A,B,..Z
- could simply try each in turn
- a **brute force search**
- given ciphertext, just try all shifts of letters
- do need to recognize when have plaintext
- eg. break ciphertext "GCUA VQ DTGCM"

MONOALPHABETIC CIPHER

- Rather than just shifting the alphabet
- could shuffle (jumble) the letters arbitrarily
- each plaintext letter maps to a different random ciphertext letter
- hence key is 26 letters long

Plain: abcdefghijklmnopqrstuvwxyz
Cipher: DKVQFIBJWPESCXHTMYAUOLRGZN

Plaintext: ifwewishtoreplaceletters
Ciphertext: WIRFRWAJUHYFTSDVFSFUUFYA

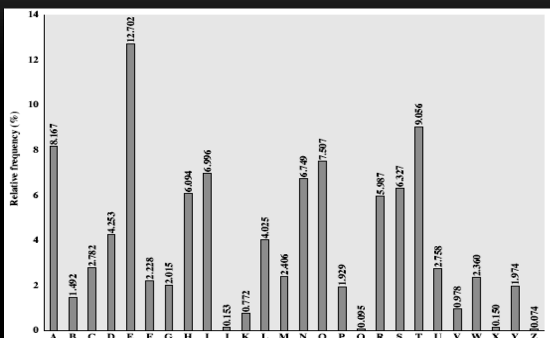
MONOALPHABETIC CIPHER SECURITY

- now have a total of 26! = 4 x 1026 keys
- with so many keys, might think is secure
- but would be **!!! WRONG!!!**
- problem is language characteristics

LANGUAGE REDUNDANCY AND CRYPTANALYSIS

- human languages are **redundant**
- eg "th lrd s m shphrd shll nt wnt"
- letters are not equally commonly used
- in English E is by far the most common letter
  - followed by T,R,N,I,O,A,S
- other letters like Z,J,K,Q,X are fairly rare
- have tables of single, double & triple letter frequencies for various languages

ENGLISH LETTER FREQUENCIES



USE IN CRYPTANALYSIS

- key concept - monoalphabetic substitution ciphers do not change relative letter frequencies
- discovered by Arabian scientists in 9th century
- calculate letter frequencies for ciphertext
- compare counts/plots against known values
- if caesar cipher look for common peaks/troughs
  - peaks at: A-E-I triple, NO pair, RST triple
  - troughs at: JK, X-Z
- for monoalphabetic must identify each letter
  - tables of common double/triple letters help

TRY TO FIND THE PLAINTEXT

- Given ciphertext:

UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESX
UDBMETSXAIZVUEPHZHMDZSHZOWSFPAPP
DTSVPQUZWYMXUZUHSXEPYEPOPDZSZUFP
OMBZWPFUPZHMDJUDTMOHMQ

EXAMPLE CRYPTANALYSIS

- Given ciphertext:

  UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ
  VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX
  EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ
- Count relative letter frequencies (see text)
- Guess P & Z are e and t
- Guess ZW is th and hence ZWP is the
- Proceeding with trial and error finally get:

  it was disclosed yesterday that several informal but direct contacts have been made with political representatives of the viet cong in moscow

PLAYFAIR CIPHER

- Not even the large number of keys in a monoalphabetic cipher provides security
- One approach to improving security was to encrypt multiple letters
- the **Playfair Cipher** is an example
- Invented by Charles Wheatstone in 1854, but named after his friend Baron Playfair

PLAYFAIR KEY MATRIX

- The best-known multiple-letter encryption cipher
- a 5X5 matrix of letters based on a keyword
- fill in letters of keyword (sans duplicates)
- fill rest of matrix with other letters
- eg. using the keyword MONARCHY

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

ENCRYPTING AND DECRYPTING PLAYFAIR CIPHER

- Plaintext is encrypted two letters at a time
  1. if a pair is a repeated letter, insert filler like 'X'
     eg. "balloon" encrypts as "ba lx lo on"
  2. if both letters fall in the same row, replace each with letter to right (wrapping back to start from end)
  3. if both letters fall in the same column, replace each with the letter below it (again wrapping to top from bottom)
  4. otherwise each letter is replaced by the letter in the same row and in the column of the other letter of the pair

SECURITY OF PLAYFAIR CIPHER

- security much improved over monoalphabetic
- since have 26 x 26 = 676 digrams
- would need a 676 entry frequency table to analyse (verses 26 for a monoalphabetic)
- and correspondingly more ciphertext
- was widely used for many years
  - eg. by US & British military in WW1
- it **can** be broken, given a few hundred letters
- since still has much of plaintext structure

POLYALPHABETIC CIPHERS

- **polyalphabetic substitution ciphers**
- improve security using multiple cipher alphabets
- make cryptanalysis harder with more alphabets to guess and flatter frequency distribution
- use a key to select which alphabet is used for each letter of the message
- use each alphabet in turn
- repeat from start after end of key is reached

VIGENÈRE CIPHER

- simplest polyalphabetic substitution cipher
- effectively multiple caesar ciphers
- key is multiple letters long $K = k_1\ k_2\ ...\ k_d$
- i^{th} letter specifies i^{th} alphabet to use
- use each alphabet in turn
- repeat from start after d letters in message
- decryption simply works in reverse

EXAMPLE OF VIGENÈRE CIPHER

- write the plaintext out
- write the keyword repeated above it
- use each key letter as a caesar cipher key
- encrypt the corresponding plaintext letter
- eg using keyword *deceptive*

  key: deceptivedeceptivedeceptive

  plaintext: wearediscoveredsaveyourself

  ciphertext:ZICVTWQNGRZGVTWAVZHCQYGLMGJ

AIDS FOR EN/DECRYPTION

- simple aids can assist with en/decryption
- a **Saint-Cyr Slide** is a simple manual aid
  - a slide with repeated alphabet
  - line up plaintext 'A' with key letter, eg 'C'
  - then read off any mapping for key letter
- can bend round into a **cipher disk**
- or expand into a **Vigenère Tableau**

SECURITY OF VIGENÈRE CIPHERS

- have multiple ciphertext letters for each plaintext letter
- hence letter frequencies are obscured
- but not totally lost
- start with letter frequencies
  - see if look monoalphabetic or not
- if not, then need to determine number of alphabets, since then can attach each

KASISKI METHOD

- Method Developed By Babbage / Kasiski
- Repetitions In Ciphertext Give Clues To Period
- So Find Same Plaintext An Exact Period Apart
- Which Results In The Same Ciphertext
- Of Course, Could Also Be Random Fluke

AUTOKEY CIPHER

- Ideally Want A Key As Long As The Message
- Vigenère Proposed The **Autokey** Cipher
- With Keyword Prefixed To Message As Key
- Knowing Keyword Can Recover The First Few Letters
- Use These In Turn On The Rest Of The Message
- But Still Have Frequency Characteristics To Attack
- Eg. Given Key *Deceptive*

  Key: Deceptivewearediscoveredsav

  Plaintext: Wearediscoveredsaveyourself

  Ciphertext:zicvtwqngkzeiigasxstslvvwla

ONE-TIME PAD

- if a truly random key as long as the message is used, the cipher will be secure
- called a One-Time pad
- is unbreakable since ciphertext bears no statistical relationship to the plaintext
- since for **any plaintext** & **any ciphertext** there exists a key mapping one to other
- can only use the key **once** though
- problems in generation & safe distribution of key

ONE TIME PAD

OTP…

- The one-time pad offers complete security but, in practice, has two fundamental difficulties:
  - *There is the practical problem of making large quantities of random keys.*
  - *And the problem of key distribution and protection, where for every message to be sent, a key of equal length is needed by both sender and receiver.*
- Because of these difficulties, the one-time pad is of limited utility, and is useful primarily for low-bandwidth channels requiring very high security.

TRANSPOSITION CIPHERS

- now consider classical **transposition** or **permutation** ciphers
- these hide the message by rearranging the letter order
- without altering the actual letters used
- can recognise these since have the same frequency distribution as the original text

RAIL FENCE CIPHER

RAIL FENCE CIPHER

- write message letters out diagonally over a number of rows
- then read off cipher row by row
- eg. write message out as:

```
m e m a t r h t g p r y
 e t e f e t e o a a t
```

- giving ciphertext

```
MEMATRHTGPRYETEFETEOAAT
```

ROW TRANSPOSITION CIPHERS

- a more complex transposition
- write letters of message out in rows over a specified number of columns
- then reorder the columns according to some key before reading off the rows

```
Key:        3 4 2 1 5 6 7
Plaintext:  a t t a c k p
            o s t p o n e
            d u n t i l t
            w o a m x y z
Ciphertext: TTNAAPTMTSUOAODWCOIXKNLYPETZ
```

PRODUCT CIPHERS

- ciphers using substitutions or transpositions are not secure because of language characteristics
- hence consider using several ciphers in succession to make harder, but:
  - two substitutions make a more complex substitution
  - two transpositions make more complex transposition
  - but a substitution followed by a transposition makes a new much harder cipher
- this is bridge from classical to modern ciphers

ROTOR MACHINES

- before modern ciphers, rotor machines were most common complex ciphers in use
- widely used in WW2
  - German Enigma, Allied Hagelin, Japanese Purple
- implemented a very complex, varying substitution cipher
- used a series of cylinders, each giving one substitution, which rotated and changed after each letter was encrypted
- with 3 cylinders have 26^3=17576 alphabets

ROTOR MACHINE…

- A rotor machine consists of a set of independently rotating cylinders through which electrical pulses can flow.
- Each cylinder has 26 input pins and 26 output pins, with internal wiring that connects each input pin to a unique output pin. If we associate each input and output pin with a letter of the alphabet, then a single cylinder defines a monoalphabetic substitution.
- After each input key is depressed, the cylinder rotates one position, so that the internal connections are shifted accordingly.
- The power of the rotor machine is in the use of multiple cylinders, in which the output pins of one cylinder are connected to the input pins of the next, and with the cylinders rotating like an "odometer", leading to a very large number of substitution alphabets being used, eg with 3 cylinders have 26^3=17576 alphabets used.
- They were extensively used in world war 2, and the history of their use and analysis is one of the great stories from WW2.

ENIGMA

HAGELIN ROTOR MACHINE

STEGANOGRAPHY

- an alternative to encryption
- hides existence of message
  - using only a subset of letters/words in a longer message marked in some way
  - using invisible ink
  - hiding in LSB in graphic image or sound file
- has drawbacks
  - high overhead to hide relatively few info bits

HILL CIPHERS

- Created by Lester S. Hill in 1929
- Polygraphic Substitution Cipher
- Uses Linear Algebra to Encrypt and Decrypt

POLYGRAPHIC SUBSTITUTION CIPHERS

- Encrypts letters in groups
- Frequency analysis more difficult

HILL CIPHERS

- Polygraphic substitution cipher
- Uses matrices to encrypt and decrypt
- Uses modular arithmetic (Mod 26)

MODULAR ARITHMETIC

- For *a* Mod *b*, divide *a* by *b* and take the remainder.
- 14 ÷ 10 = 1 R 4
- 14 Mod 10 = 4
- 24 Mod 10 = 4

MODULUS THEOREM

THEOREM:
For an integer a and modulus m, let

$$R = \text{remainder of } \frac{|a|}{m}$$

Then the residue r of a modulus m is given by:

$$r = \begin{cases} R & if \quad a \geq 0 \\ m - R & if \quad a < 0 \quad and \quad R \neq 0 \\ 0 & if \quad a < 0 \quad and \quad R = 0 \end{cases}$$

MODULUS EXAMPLES

1. 80 Mod 26

$$\frac{|80|}{26} = 3 \text{ R } 2 \Rightarrow 80 \geq 0 \Rightarrow 80 \text{ Mod } 26 = 2$$

2. -80 Mod 26

$$\frac{|-80|}{26} = 3 \text{ R } 2 \Rightarrow -80 < 0 \text{ and } 2 \neq 0 \Rightarrow -80 \text{ Mod } 26 = 26 - 2 = 24$$

3. -52 Mod 26

$$\frac{|-52|}{26} = 2 \text{ R } 0 \Rightarrow -52 < 0 \text{ and } 0 = 0 \Rightarrow -52 \text{ Mod } 26 = 0$$

MODULAR INVERSES

- Inverse of 2 is ½ (2 · ½ = 1)
- Matrix Inverse: $AA^{-1} = I$
- Modular Inverse for Mod m: $(a \cdot a^{-1})$ Mod $m = 1$
- For Modular Inverses, *a* and *m* must NOT have any prime factors in common

MODULAR INVERSES OF MOD 26

A	1	2	5	7	9	11	15	17	19	21	23	25
A^{-1}	1	9	21	15	3	19	7	23	11	5	17	25

Example – Find the Modular Inverse of 9 for Mod 26

$9 \cdot 3 = 27$

27 Mod 26 = 1

3 is the Modular Inverse of 9 Mod 26

HILL CIPHER MATRICES

- One matrix to encrypt, one to decrypt
- Must be n x n, invertible matrices
- Decryption matrix must be modular inverse of encryption matrix in Mod 26

MODULARLY INVERSE MATRICES

- Calculate determinant of first matrix A, det A
- Make sure that det A has a modular inverse for Mod 26
- Calculate the adjugate of A, adj A
- Multiply adj A by modular inverse of det A
- Calculate Mod 26 of the result to get B
- Use A to encrypt, B to decrypt

MODULAR RECIPROCAL EXAMPLE

Let $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

det A = (2 x 4) - (1 x 3) = 8 - 3 = 5

modular inverse of 5 for Mod 26 = 21

$$B = 21\begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 84 & -21 \\ -63 & 42 \end{bmatrix}$$

$$B = \begin{bmatrix} 84 & -21 \\ -63 & 42 \end{bmatrix} \text{ Mod } 26 = \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}$$

Therefore $\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}$ is the modular inverse of $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ for Mod 26. $\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}$

ENCRYPTION

- Assign each letter in alphabet a number between 0 and 25
- Change message into 2 x 1 letter vectors
- Change each vector into 2 x 1 numeric vectors
- Multiply each numeric vector by encryption matrix
- Convert product vectors to letters

LETTER TO NUMBER SUBSTITUTION

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

CHANGE MESSAGE TO VECTORS

Message to encrypt = HELLO WORLD

$$\begin{bmatrix} H \\ E \end{bmatrix} \quad \begin{bmatrix} L \\ L \end{bmatrix} \quad \begin{bmatrix} O \\ W \end{bmatrix} \quad \begin{bmatrix} O \\ R \end{bmatrix} \quad \begin{bmatrix} L \\ D \end{bmatrix}$$

$$\begin{bmatrix} H \\ E \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} L \\ L \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} O \\ W \end{bmatrix} = \begin{bmatrix} 14 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} O \\ R \end{bmatrix} = \begin{bmatrix} 14 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} L \\ D \end{bmatrix} = \begin{bmatrix} 11 \\ 3 \end{bmatrix}$$

MULTIPLY MATRIX BY VECTORS

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \text{x} \begin{bmatrix} 7 \\ 4 \end{bmatrix} = \begin{bmatrix} 14+4 \\ 21+16 \end{bmatrix} = \begin{bmatrix} 18 \\ 37 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \text{x} \begin{bmatrix} 11 \\ 11 \end{bmatrix} = \begin{bmatrix} 22+11 \\ 33+44 \end{bmatrix} = \begin{bmatrix} 33 \\ 77 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \text{x} \begin{bmatrix} 14 \\ 22 \end{bmatrix} = \begin{bmatrix} 28+22 \\ 42+88 \end{bmatrix} = \begin{bmatrix} 50 \\ 130 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \text{x} \begin{bmatrix} 14 \\ 17 \end{bmatrix} = \begin{bmatrix} 28+17 \\ 42+68 \end{bmatrix} = \begin{bmatrix} 45 \\ 110 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \text{x} \begin{bmatrix} 11 \\ 3 \end{bmatrix} = \begin{bmatrix} 22+3 \\ 33+12 \end{bmatrix} = \begin{bmatrix} 25 \\ 45 \end{bmatrix}$$

CONVERT TO MOD 26

$$\begin{bmatrix} 18 \\ 37 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 18 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 33 \\ 77 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 7 \\ 25 \end{bmatrix}$$

$$\begin{bmatrix} 50 \\ 130 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 24 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 45 \\ 110 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 19 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 25 \\ 45 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 25 \\ 19 \end{bmatrix}$$

CONVERT NUMBERS TO LETTERS

$$\begin{bmatrix} 18 \\ 11 \end{bmatrix} = \begin{bmatrix} S \\ L \end{bmatrix}$$

$$\begin{bmatrix} 7 \\ 25 \end{bmatrix} = \begin{bmatrix} H \\ Z \end{bmatrix}$$

$$\begin{bmatrix} 24 \\ 0 \end{bmatrix} = \begin{bmatrix} Y \\ A \end{bmatrix}$$

$$\begin{bmatrix} 19 \\ 6 \end{bmatrix} = \begin{bmatrix} T \\ G \end{bmatrix}$$

$$\begin{bmatrix} 25 \\ 19 \end{bmatrix} = \begin{bmatrix} Z \\ T \end{bmatrix}$$

HELLO WORLD has been encrypted to SLHZY ATGZT

DECRYPTION

- Change message into 2 x 1 letter vectors
- Change each vector into 2 x 1 numeric vectors
- Multiply each numeric vector by decryption matrix
- Convert new vectors to letters

CHANGE MESSAGE TO VECTORS

Message to encrypt = SLHZYATGZT

$$\begin{bmatrix} S \\ L \end{bmatrix} \quad \begin{bmatrix} H \\ Z \end{bmatrix} \quad \begin{bmatrix} Y \\ A \end{bmatrix} \quad \begin{bmatrix} T \\ G \end{bmatrix} \quad \begin{bmatrix} Z \\ T \end{bmatrix}$$

$$\begin{bmatrix} S \\ L \end{bmatrix} = \begin{bmatrix} 18 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} H \\ Z \end{bmatrix} = \begin{bmatrix} 7 \\ 25 \end{bmatrix}$$

$$\begin{bmatrix} Y \\ A \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} T \\ G \end{bmatrix} = \begin{bmatrix} 19 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} Z \\ T \end{bmatrix} = \begin{bmatrix} 25 \\ 19 \end{bmatrix}$$

MULTIPLY MATRIX BY VECTORS

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix} \text{x} \begin{bmatrix} 18 \\ 11 \end{bmatrix} = \begin{bmatrix} 108+55 \\ 270+176 \end{bmatrix} = \begin{bmatrix} 163 \\ 446 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix} \text{x} \begin{bmatrix} 7 \\ 25 \end{bmatrix} = \begin{bmatrix} 42+125 \\ 105+400 \end{bmatrix} = \begin{bmatrix} 167 \\ 505 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix} \text{x} \begin{bmatrix} 24 \\ 0 \end{bmatrix} = \begin{bmatrix} 144+0 \\ 360+0 \end{bmatrix} = \begin{bmatrix} 144 \\ 360 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix} \text{x} \begin{bmatrix} 19 \\ 6 \end{bmatrix} = \begin{bmatrix} 114+30 \\ 285+96 \end{bmatrix} = \begin{bmatrix} 144 \\ 381 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix} \text{x} \begin{bmatrix} 25 \\ 19 \end{bmatrix} = \begin{bmatrix} 150+95 \\ 375+304 \end{bmatrix} = \begin{bmatrix} 245 \\ 679 \end{bmatrix}$$

CONVERT TO MOD 26

$$\begin{bmatrix} 163 \\ 446 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 167 \\ 505 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 11 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 144 \\ 360 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 14 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} 144 \\ 381 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 14 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} 245 \\ 679 \end{bmatrix} \text{Mod } 26 = \begin{bmatrix} 11 \\ 3 \end{bmatrix}$$

CONVERT NUMBERS TO LETTERS

$$\begin{bmatrix} 7 \\ 4 \end{bmatrix} = \begin{bmatrix} H \\ E \end{bmatrix}$$

$$\begin{bmatrix} 11 \\ 11 \end{bmatrix} = \begin{bmatrix} L \\ L \end{bmatrix}$$

$$\begin{bmatrix} 14 \\ 22 \end{bmatrix} = \begin{bmatrix} O \\ W \end{bmatrix}$$

$$\begin{bmatrix} 14 \\ 17 \end{bmatrix} = \begin{bmatrix} O \\ R \end{bmatrix}$$

$$\begin{bmatrix} 11 \\ 3 \end{bmatrix} = \begin{bmatrix} L \\ D \end{bmatrix}$$

SLHZYATGZT has been decrypted to HELLO WORLD

CONCLUSION

- Creating valid encryption/decryption matrices is the most difficult part of Hill Ciphers.
- Otherwise, Hill Ciphers use simple linear algebra and modular arithmetic

SUMMARY

- have considered:
  - classical cipher techniques and terminology
  - monoalphabetic substitution ciphers
  - cryptanalysis using letter frequencies
  - Playfair cipher
  - polyalphabetic ciphers
  - transposition ciphers
  - product ciphers and rotor machines
  - stenography