Unit 3: Transistor at Low Frequencies

BJT Transistor Modeling

- A model is an equivalent circuit that represents the AC characteristics of the transistor.
- A model uses circuit elements that approximate the behavior of the transistor.
- There are two models commonly used in small signal AC analysis of a transistor:
  - $r_e$ model
  - Hybrid equivalent model

The $r_e$ Transistor Model

- BJTs are basically current-controlled devices; therefore the $r_e$ model uses a diode and a current source to duplicate the behavior of the transistor.
- One disadvantage to this model is its sensitivity to the DC level. This model is designed for specific circuit conditions.

Common-Base Configuration
\[ I_c = \alpha I_e \]
\[ r_e = \frac{26 \text{ mV}}{I_c} \]

input impedance: \[ Z_i = r_e \]

Output impedance: \[ Z_o \cong \infty \Omega \]

Voltage gain: \[ A_v = \frac{\alpha r_e}{r_e} \cong \frac{R_L}{r_e} \]

Current gain: \[ A_i = -\alpha \cong -1 \]

**Common-Emitter Configuration**

The diode \( r_e \) model can be replaced by the resistor \( r_e \).

\[ I_e = (\beta + 1)I_b \cong \beta I_b \]

\[ r_e = \frac{26 \text{ mV}}{I_c} \]
Input impedance:

\[ Z_i = \beta r_e \]

Output impedance:

\[ Z_o = r_o \cong \infty \Omega \]

Voltage gain:

\[ A_V = \frac{R_L}{r_e} \]

Current gain:

\[ A_i = \beta \mid_{r_o=\infty} \]

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**Common-Collector Configuration**

Input impedance:

\[ Z_i = (\beta + 1)r_e \]

Output impedance:

\[ Z_o = r_e \parallel R_E \]

Voltage gain:

\[ A_V = \frac{R_E}{R_E + r_e} \]

Current gain:

\[ A_i = \beta + 1 \]
The Hybrid Equivalent Model

The following hybrid parameters are developed and used for modeling the transistor. These parameters can be found on the specification sheet for a transistor.

- \( h_i \) = input resistance
- \( h_r \) = reverse transfer voltage ratio \( (V_i/V_o) \equiv 0 \)
- \( h_f \) = forward transfer current ratio \( (I_o/I_i) \)
- \( h_o \) = output conductance

Simplified General h-Parameter Model

- \( h_i \) = input resistance
- \( h_f \) = forward transfer current ratio \( (I_o/I_i) \)
The Hybrid p Model

The hybrid p model is most useful for analysis of high-frequency transistor applications.

At lower frequencies the hybrid p model closely approximate the $r_e$ parameters, and can be replaced by them.

Common-Emitter Fixed-Bias Configuration

- The input is applied to the base
- The output is from the collector
- High input impedance
- Low output impedance
- High voltage and current gain
- Phase shift between input and output is $180^\circ$
Common-Emitter Fixed-Bias Configuration

Input impedance:
\[ Z_i = R_B || \beta r_e \]
\[ Z_i \approx \beta r_e \quad R_e \geq 10/\beta \]

Output impedance:
\[ Z_o = R_C || r_o \]
\[ Z_o \approx R_C \quad r_e \geq 10 R_C \]

Voltage gain:
\[ A_v = \frac{V_o}{V_i} = -\frac{(R_C || r_o)}{r_e} \]
\[ A_v = -\frac{R_C}{r_e} \quad r_e \geq 10 R_C \]

Current gain:
\[ A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)} \]
\[ A_i \approx \beta \quad r_e \geq 10 R_C, R_B \geq 10 \beta r_e \]
Current gain from voltage gain:

\[ A_i = -A_v \frac{Z_i}{R_C} \]

Common-Emitter Voltage-Divider Bias

Calaculations:

Input impedance:

\[ R' = R_1 \parallel R_2 \]
\[ Z_i = R' \parallel /\beta r_e \]

Output impedance:

\[ Z_o = R_C \parallel r_o \]
\[ Z_o \approx R_C \bigg|_{r_o \geq 10R_C} \]

Voltage gain:

\[ A_v = \frac{V_o}{V_i} = -\frac{R_C \parallel r_o}{r_e} \]
\[ A_v = \frac{V_o}{V_i} \approx -\frac{R_C}{r_e} \bigg|_{r_o \geq 10R_C} \]

The Re model requires you to determine \( \beta, r_e, \) and \( r_o \).
Current gain:

\[
A_i = \frac{I_o}{I_i} = \frac{\beta R' r_o}{(r_o + R_c) (R' + \beta r_e)}
\]

\[
A_i = \frac{I_o}{I_i} = \frac{\beta R'}{R' + \beta r_e} \bigg|_{r_o \gg R_c, R' \gg \beta r_e}
\]

\[
A_i = \frac{I_o}{I_i} \approx \beta \bigg|_{r_o \gg R_c, R' \gg \beta r_e}
\]

Current gain from voltage gain:

\[
A_i = -A_v \frac{Z_i}{R_C}
\]

Common-Emitter Emitter-Bias Configuration

Impedance Calculations

**Input impedance:**

\[
Z_i = R_B \parallel Z_b
\]

\[
Z_b = \beta r_e + (\beta + 1) R_E
\]

\[
Z_b \approx \beta (r_e + R_E)
\]

\[
Z_b \approx \beta R_E
\]

**Output impedance:**

\[
Z_o = R_C
\]
Gain Calculations

Voltage gain: \[ A_v = \frac{v_o}{v_i} = -\frac{\beta R_C}{Z_b} \]
\[ A_v = \frac{v_o}{v_i} = -\frac{R_C}{r_c + R_E} \left| z_o = \beta (r_e + R_E) \right| \]
\[ A_v = \frac{v_o}{v_i} \approx -\frac{R_C}{R_E} \left| z_o = \beta R_E \right| \]

Current gain:
\[ A_i = \frac{i_o}{i_i} = \frac{\beta R_B}{R_B + Z_b} \]

Current gain from voltage gain:
\[ A_i = -A_v \frac{Z_o}{R_C} \]

Emitter-Follower Configuration

- This is also known as the common-collector configuration.
- The input is applied to the base and the output is taken from the emitter.

There is no phase shift between input and output.
Impedance Calculations

Input impedance:

\[ Z_i = R_B \parallel Z_b \]
\[ Z_b = \beta r_e + (\beta + 1)R_E \]
\[ Z_b \approx \beta (r_e + R_E) \]
\[ Z_b \approx \beta R_E \]

Output impedance:

\[ Z_o = R_E \parallel r_e \]
\[ Z_o \approx r_e \bigg|_{R_E \gg r_e} \]

Gain Calculations

Voltage gain:

\[ A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e} \]
\[ A_v = \frac{V_o}{V_i} \approx 1 \bigg|_{r_e \gg r_i, R_E \gg r_i} \]

Current gain:

\[ A_i \approx -\frac{\beta R_B}{R_B + Z_b} \]
Current gain from voltage gain:

\[ A_i = -A_v \frac{Z_i}{R_E} \]

**Common-Base Configuration**

- The input is applied to the emitter.
- The output is taken from the collector.
- Low input impedance.
- High output impedance.
- Current gain less than unity.
- Very high voltage gain.
- No phase shift between input and output.

**Calculations**

**Input impedance:**

\[ Z_i = R_E \parallel r_e \]
Output impedance: $Z_o = R_C$

Voltage gain:
$$A_v = \frac{V_o}{V_i} = \frac{\alpha R_C}{r_e} \approx \frac{R_C}{r_e}$$

Current gain:
$$A_i = \frac{I_o}{I_i} = -\alpha \approx -1$$

Common-Emitter Collector Feedback Configuration

- This is a variation of the common-emitter fixed-bias configuration
- Input is applied to the base
- Output is taken from the collector
- There is a $180^\circ$ phase shift between input and output

Calculations

Input impedance:
$$Z_i = \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_F}}$$
Output impedance: \[ Z_o \approx R_C \parallel R_F \]

Voltage gain:

\[ A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e} \]

Current gain:

\[ A_i = \frac{I_o}{I_i} = \frac{\beta R_F}{R_F + \beta R_C} \]

\[ A_i = \frac{I_o}{I_i} \approx \frac{R_F}{R_C} \]

Collector DC Feedback Configuration

- This is a variation of the common-emitter, fixed-bias configuration
- The input is applied to the base
- The output is taken from the collector
- There is a 180° phase shift between input and output
Calculations

Input impedance:
\[ Z_i = \frac{r_c}{1 + \frac{R_C}{\beta R_F}} \]

Output impedance:
\[ Z_o \cong R_C \parallel R_F \]

Voltage gain:
\[ A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_c} \]

Current gain:
\[ A_i = \frac{I_o}{I_i} = \frac{\beta R_F}{R_F + \beta R_C} \]
\[ A_i \cong \frac{R_F}{R_C} \]

Two-Port Systems Approach
This approach:

- Reduces a circuit to a two-port system
- Provides a “Thévenin look” at the output terminals
- Makes it easier to determine the effects of a changing load

With $V_i$ set to 0 V: $Z_{Th} = Z_o = R_o$

The voltage across the open terminals is:

$$E_{Th} = A_{vNL} V_i$$

where $A_{vNL}$ is the no-load voltage gain.

### Effect of Load Impedance on Gain

This model can be applied to any current- or voltage-controlled amplifier.

Adding a load reduces the gain of the amplifier:

$$A_v = \frac{V_o}{V_i} = \frac{R_L}{R_L + R_o} A_{vNL}$$

$$A_i = -A_v \frac{Z_i}{R_L}$$
The fraction of applied signal that reaches the input of the amplifier is:

\[ V_i = \frac{R_i V_s}{R_i + R_s} \]

The internal resistance of the signal source reduces the overall gain:

\[ A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} A_{vNL} \]

**Combined Effects of \( R_s \) and \( R_L \) on Voltage Gain**

Effects of \( R_L \):

\[ A_v = \frac{V_o}{V_i} = \frac{R_L A_{vNL}}{R_L + R_o} \]

\[ A_i = -A_v \frac{R_i}{R_L} \]
Effects of $R_L$ and $R_S$:

$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} \frac{R_L}{R_L + R_o} A_{vNL}$$

$$A_{is} = -A_{vs} \frac{R_s + R_i}{R_L}$$

**Cascaded Systems**

The output of one amplifier is the input to the next amplifier.

The overall voltage gain is determined by the product of gains of the individual stages.

The DC bias circuits are isolated from each other by the coupling capacitors.

The DC calculations are independent of the cascading.

The AC calculations for gain and impedance are interdependent.

**R-C Coupled BJT Amplifiers**

**Input impedance, first stage:**

$$Z_i = R_1 \parallel R_2 \parallel \beta r_e$$

**Output impedance, second stage:**

$$Z_o = R_C$$
Voltage gain:

\[ A_{v1} = \frac{R_C \parallel R_i \parallel R_2 \parallel \beta r_e}{r_e} \]

\[ A_{v2} = \frac{R_C}{r_e} \]

\[ A_v = A_{v1} A_{v2} \]